SRI CHANDRASEKHARENDRA SARASWATHI VISWA MAHAVIDYALAYA

(University established under section 3 of UGC Act 1956)
(Accredited with 'A' Grade by NAAC)
Enathur, Kanchipuram – 631 561

Course Material

SUBJECT	:	**NETWORK THEORY**
YEAR/SEM	:	**II/II**
DEPARTMENT	:	**ECE**

Prepared by,

Dr.A.RAJASEKARAN,
Assistant Professor
Department of Electronics and Communication Engineering
SRI CHANDRASEKHARENDRA SARASWATHI VISWA MAHAVIDYALAYA
ENATHUR, KANCHIPURAM - 631561

NETWORK THEORY

Prerequisite: Mathematics – II & Basic Electronics Engineering

OBJECTIVES:

- To learn techniques of solving circuits involving different active and passive elements.
- To analyze the behavior of the circuit's response in time domain.
- To analyze the behavior of the circuit's response in frequency domain.
- To understand the significance of network function

UNIT-1 (CIRCUIT ANALYSIS)

KVL- KCL- circuit elements(R,L &C) in series and parallel- voltage and current divider rule-source transformation technique-duals and duality- mesh analysis-super mesh analysis-nodal analysis-super nodal analysis-network topology-definitions-incident matrix-fundamental cut set matrix-series and parallel resonance.

UNIT-2(NETWORK THEOREMS)

Superposition theorem, Thevenin's theorem, Norton's theorem, Maximum power Transfer theorem, reciprocity theorem, compensation theorem, and Tallegen's theorem as applied to DC and AC. Circuits

UNIT-3(TWO PORT NETWORKS AND FILTERS DESIGN)

Z parameter, Y parameter, h parameter, ABCD parameter, g parameter,Inter relationship of different parameters-inter connection of two port networks-classification of filters-constant k low pass and high pass filters-m-derived low pass and high pass filters-band pass filter-band elimination filter .

UNIT-4(TRANSIENT AND S-DOMAIN ANALYSIS)

Steady state and transient response-DC response of an R-L,R-C and R-L-C circuit-sinusoidal response of R-L,R-C and R-L-C circuit-concept of complex of frequency-poles and zeros of network function-significance of poles and zeros-properties of driving point and transfer function.

UNIT-5(NETWORK SYNTHESIS)

Hurwitz polynomial-positive real function, frequency response of reactive one port-synthesis of reactive one port by Foster's Method &Cauer method- synthesis of R-L Network by Foster's Method &Cauer method- synthesis of R-C Network by Foster's Method &Cauer method.

OUTCOMES:

- Understand the behavior of different circuits and their response using various circuit analysis tools and theorems
- Understand the analysis in time domain and frequency domain.
- Understand basic concepts regarding the system definition mathematically and associated network function.
- Understand the concept of Network synthesis.

Text Books:

1. Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi, 1994
2. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education.

Unit I -NETWORK ANALYSIS

AIM:

To create circuits involving different active and passive elements

Pre-Requisites:

Knowledge of Basic Mathematics – II & Basic Electronics Engineering

Pre - MCQs:

1. **Time constant of a capacitive circuit**
   a. Increases with the decrease of capacitance and decrease of resistance
   b. Increases with the decrease of capacitance and increase of resistance
   c. Increases with the increase of capacitance and decrease of resistance
   d. Increase with increase of capacitance and increase of resistance

2. **Which of the following is a correct statement of Ohm's law?**
   a. I = R/V
   b. R = VI
   c. V = I/R
   d. I = V/R

3. **A sinusoidal signal has a period of 40 ms. What is its frequency?**
   a. 25 Hz
   b. 50 kHz
   c. 50 Hz
   d. 25 kHz

4. **In a loss-free R-L-C circuit the transient current is**
   a. Oscillating
   b. Square wave
   c. Sinusoidal
   d. Non-oscillating

5. **In a circuit containing R, L and C, power loss can take place in**
   a. C only
   b. L only
   c. R only
   d. All the above

NETWORK ANALYSIS USING KVL & KCL

INTRODUCTION:

Today we live in a predominantly electrical world. Electrical technology is a driving force in the changes that are occurring in every engineering discipline. Circuit analysis is the foundation for electrical technology.Network is a system with interconnected electrical elements. Network and circuit are the same. The only difference being a circuit shall contain at least one closed path.

Network analysis is the process of finding the voltages across, and the currents through every component in the network.

Basic Circuit Elements

Circuit: A circuit is a closed conducting path through which an electric current flows.

Electric Network: A combination of various electric elements, connected in any manner is called an electric network.

Electric Circuits consist of two basic types of elements. These are the *active elements* and the *passive elements*.
An ***active element*** is capable of generating or supplying an electrical energy.
Examples are *voltage source* (such as a battery or generator) and *current source*, oscillators etc.. A ***passive element*** is one which does not generate electricity but either consumes it or stores it. Resistors, Inductors and Capacitors are simple passive elements. Diodes, transistors etc. are also passive elements.

These parameters may be *lumped* or *distributed.*
Elements of a circuit, which are separated physically, are known as ***lumped elements***.
Ex:- L & C.

Elements, which are not separable for analytical purposes, are known as ***distributed elements***.
Ex:- Transmission lines having R, L, C all along their length.

Circuits may either be *linear* or *non-linear*
A ***linear circuit*** is one whose parameter are constant i.e., they do not change with voltage or current. Linear elements obey a straight line law.

For example, a linear resistor has a linear *voltage v/s current* relationship which passes through the origin (V = R.I). A linear inductor has a linear *flux vs current* relationship which passes through the origin (ϕ = LI) and a linear capacitor has a linear *charge vs voltage* relationship which passes through the origin (q = CV). [R, L and C are constants].

A ***Non linear circuit*** is one whose parameters change with voltage or current.

Resistors, inductors and capacitors may be linear or non-linear, while diodes and transistors are always nonlinear.

Circuits may either be *Unilateral* or *Bilateral*
The circuit whose properties or characteristics change with the direction of its operation is said to be ***Unilateral***. A diode rectifier is a unilateral, because it cannot perform rectification in both directions.

A ***bilateral circuit*** is one whose properties or characteristics are the same in either direction. Examples are R, L & C. The usual transmission line is bilateral, because it can be made to perform its function equally well in either direction.

Branch

A *branch* represents a single element, such as a resistor or a battery. A branch is a part of the network which lies between two junctions

Node

A *node* is the point or junction in a circuit connecting two or more branches or circuit elements. The node is usually indicated by a dot (**.**) in a circuit

Loop

A *loop* is any closed path in a circuit, formed by starting at a node, passing through a number of branches and ending up once more at the original node. No element or node is encountered more than once.

Mesh

It is a loop that contains no other loop within it.

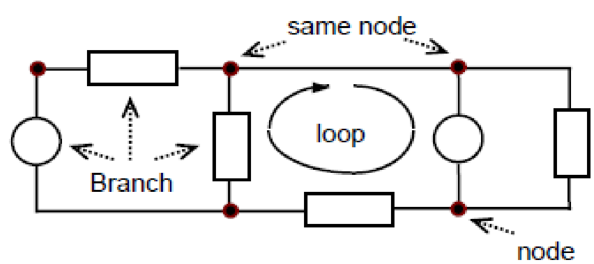
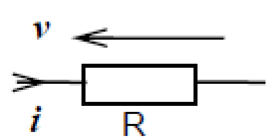


Fig.1.1

For example, the circuit of Fig 1.1 has 4 nodes, 6 branches and 6 loops and 3 meshes.

Resistance R [Unit: Ohm (Ω)]



The relationship between voltage and current is given by $v = \mathbf{R}\, i$, or $i = G\, v$,
G = conductance = 1/R

Power loss in a resistor = $R\,i^2$. Energy dissipated in a resistor $w = \int R.i^2\,dt$
There is no storage of energy in a resistor.

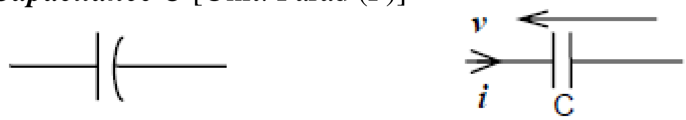
***Inductance L** [Unit: Henry (H)]*

The relationship between voltage and current is given by $\mathbf{v = N\frac{d\emptyset}{dt}=L\frac{di}{dt}}$

Energy stored in an inductor = $\frac{1}{2} L\, i^2$

No energy is dissipated in a pure inductor. However as practical inductors have some wire resistance there would be some power loss. There would also be a small power loss in the magnetic core (if any).

Capacitance C [Unit: Farad (F)]



The relationship between voltage and current is given by $i = \frac{dq}{dt} = C\frac{dv}{dt}$

Energy stored in an capacitor = $\frac{1}{2} C\, v^2$

No energy is dissipated in a pure capacitor. However practical capacitors also have some power loss.

Independent and Dependent sources

Those voltage or current sources, which do not depend on any other quantity in the circuit, are called independent sources. An independent d.c. voltage source is shown in Fig.1.2 (a) whereas a time varying voltage source is shown in Fig.1.2 (b). The positive sign shows that terminal A is positive with respect to terminal B. In other words, potential of terminal A is v volts higher than the terminal B.

Similarly, Fig.1.2 (c) shows an ideal constant current source whereas Fig.1.2 (d) depicts a time-varying current source. The arrow shows the direction of flow of the current at any moment under consideration.

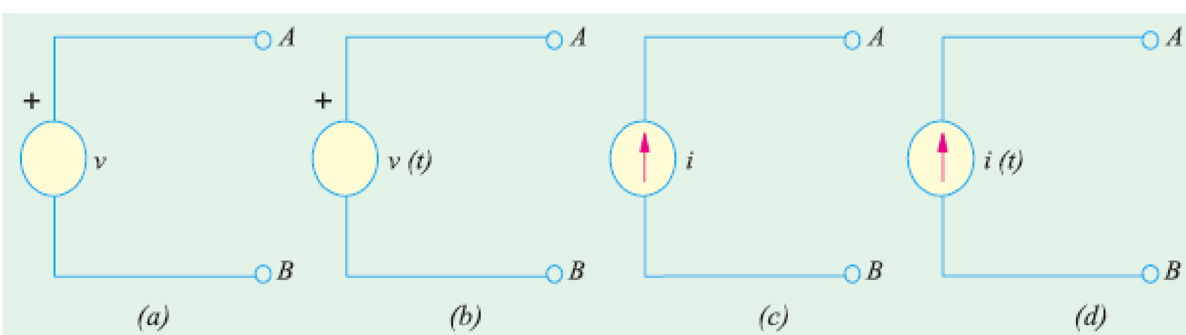


Fig.1.2: Independent voltage and current sources

A dependent voltage or current source is one which depends on some other quantity which may be either a voltage or a current. Such a source is represented by a diamond shape as shown in Fig.1.3. There are four possible dependent sources.

1. Voltage-dependent voltage source [Fig.1.3 (a)]
2. Current- dependent voltage source [Fig.1.3 (b)]
3. Voltage- dependent current source [Fig.1.3 (c)]
4. Current - dependent current source [Fig.1.3 (d)]

Such sources can also be either constant sources or time-varying sources. The constant of proportionality are written as α,r,g and β. The constants α and β have no units, r has the unit of ohms and g has the unit of seimens.

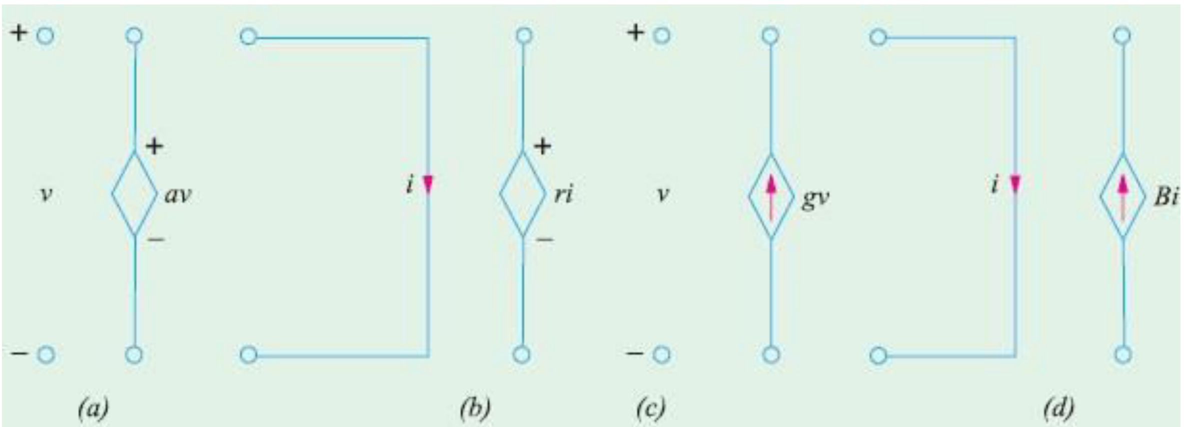


Fig.1.3: Dependent voltage and current sources

Fundamental Laws

The fundamental laws that govern electric circuits are Ohm’s law and Kirchoff’s laws.

Ohm’s Law

Ohm’s law states that the voltage **v** across a resistor is directly proportional to the current **i** flowing through it.

$v \propto i$, $v = R \cdot i$ where R is the proportionality constant.

A ***short circuit*** in a circuit element is when the resistance (and any other impedance) of the element approaches zero. [The term impedance is similar to resistance but is used in alternating current theory for other components]

An ***open circuit*** in a circuit element is when the resistance (and any other impedance) of the element approaches infinity.

In addition to Ohm’s law we need the Kirchoff’s voltage law and the Kirchoff’s current law to analyse circuits.

Kirchoff's Current Law

Kirchoff's Current Law states that the algebraic sum of the currents entering a node is zero. It simply means that the total current leaving a junction is equal to the current entering that junction.

$$\Sigma i = 0$$

Consider the case of a few conductors meeting at a point A as in Fig.1.4. Some conductors have currents leading to point A, whereas some have currents leading away from point A.

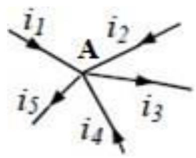


Fig.1.4

Assuming the incoming currents to be positive and the outgoing currents negative, we have
$i_1 + i_2 - i_3 + i_4 - i_5 = 0$

Kirchoff's Voltage Law

Kirchoff's Voltage Law states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\Sigma v = 0$$

In other words, $\sum IR + \sum e.m.f. = 0$ ... round a mesh

Consider a circuit as shown in Fig.1.5, we have
$$-v1 + v2 + v3 + v4 = 0$$
depending on the convention, you may also write
$$v1 - v2 - v3 - v4 = 0$$

Note: v1, v2 … may be voltages across either active elements or passive elements or both and may be obtained using Ohm's law.

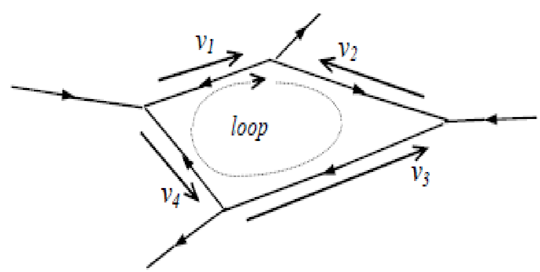


Fig.1.5

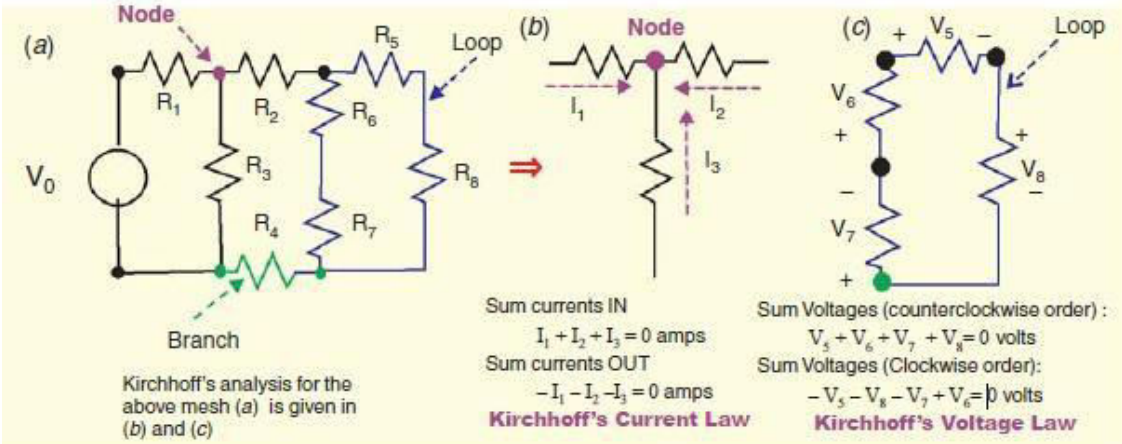


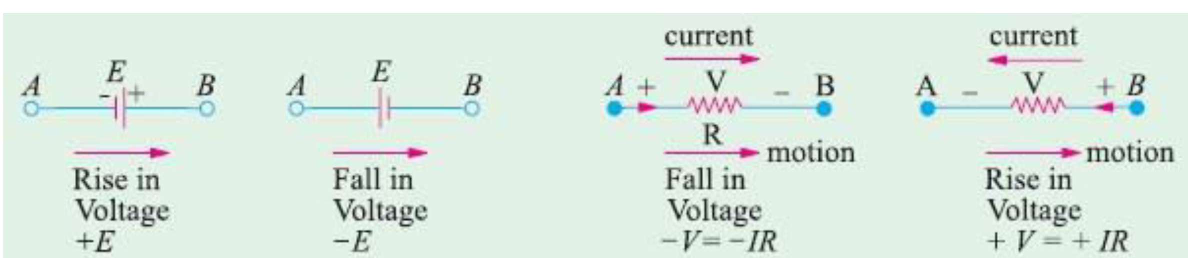
Fig.1.6: Kirchoff's analysis circuit

Determination of Voltage sign

In applying Kirchhoff's laws to specific problems, for example, the circuit shown in Fig.1.6, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs. Following sign conventions is suggested.

(a) Sign of Battery E.M.F.

A rise in voltage should be given a +ve sign and a fall in voltage a –ve sign. Keeping this in mind, it is clear that as we go from the –ve terminal of a battery to its +ve terminal as shown in Fig.1.7(a) there is a rise in potential, hence this voltage should be given a +ve sign. On the other hand, if we go from the +ve terminal of a battery to its -ve terminal) there is a fall in potential, hence this voltage should be preceeded by a -ve sign. It is important to note that the sign of the battery e.m.f is independent of the direction of the current through that branch.



(a) (b)

Fig.1.7: Voltage Sign

(b) Sign of IR Drop

Now, take the case of a resistor for Fig.1.7 (b). If we go through a resistor in the same direction as of the current, then there is a fall in potential because current flows from a higher to lower potential. Hence this voltage fall should be taken –ve. However, if we go in a direction opposite of the current, then there is a rise in voltage. Hence this voltage rise should be given a +ve.

Consider the closed path ABCDA in Fig.1.8, as we travel around the mesh in clockwise direction, using KVL we get,

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

Or $$I_1R_1 + I_2R_2 + I_3R_3 + I_4R_4 = E_1 - E_2$$

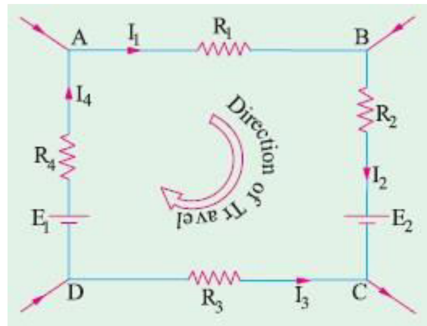


Fig.1.8

Assumed Direction of Current

The direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of the current is not actual direction, then on solving the question, this current will be found to have a minus sign. If the answer is positive, then assumed direction is same as actual direction.

Voltage Divider

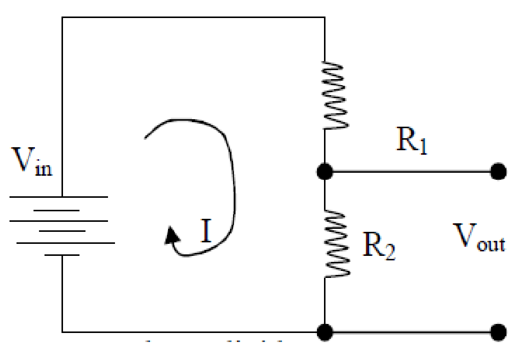


Fig.1.9: Voltage Divider

The voltage divider circuit is shown in Fig.1.9.

Ohm's law gives $V_{out} = IR_2$ …… (1)

and we know that $I = \frac{Vin}{Re} = \frac{V_{in}}{R_1 + R_2}$……. (2)

Substituting eqn (2) into eqn (1) gives eqn (3)

$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$……….(3)

In general, if there are n resistors in series, the voltage across resistor Rx is given by

$V_X = V_{in} \times \frac{R_X}{R_1 + R_2 + R_3 \dots + R_n}$

Current Divider

The two-resistor circuits shown in the circuit Fig.1.10 is a current divider circuit. The current through R_1 is given by,

$I_1 = I_t \times \frac{R_S}{R_1 + R_S}$

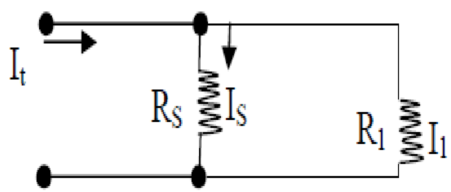
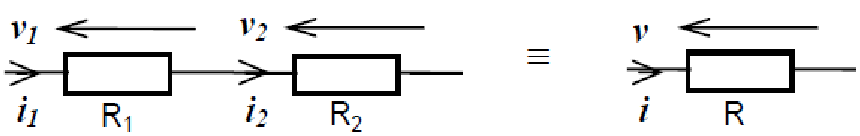


Fig.1.10: Current Divider

Series Circuits



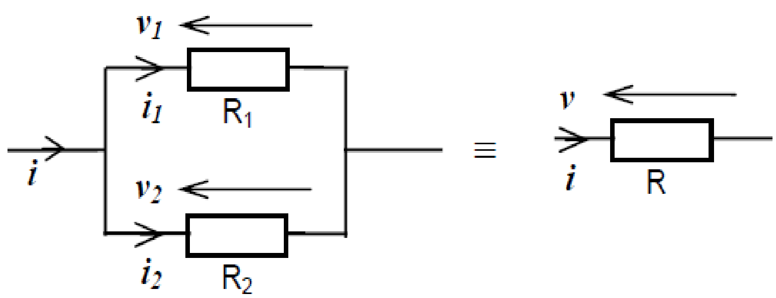
When elements are connected in series, from Kirchoff's current law, $\boldsymbol{i_1 = i_2 = i}$ and from Kirchoff's Voltage Law, $\boldsymbol{v_1 + v_2 = v}$. Also from Ohm's Law,

$v_1 = R_1 i_1$, $v_2 = R_2 i_2$, $v = R I$

$\therefore R_1 i + R_2 i = R i$, or $R = R_1 + R_2$

That is, in a series circuit, the total resistance is the sum of the individual resistances, and the voltage across the individual elements is directly proportional to the resistance of that element.

Parallel Circuits



When elements are connected in parallel, from Kirchoff's current law, $i_1 + i_2 = i$ and from Kirchoff's Voltage Law, $v_1 = v_2 = v$. Also from Ohm's Law, $v_1 = R_1\, i_1$, $v_2 = R_2\, i_2$, $v = R\, i$

$$\therefore \frac{v}{R_1}+\frac{v}{R_2}=\frac{v}{R} \quad or \quad \frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \quad or \; R=\frac{R_1 R_2}{R_1+R_2}$$

$$Also, \frac{i_1}{i_2}=\frac{\frac{v_1}{R_1}}{\frac{v_2}{R_2}}=\frac{R_2\, v}{R_1\, v}=\frac{R_2}{R_1} \quad and \frac{i_1}{i}=\frac{R_2}{R_1+R_2}, \frac{i_2}{i}=\frac{R_1}{R_1+R_2}$$ .... current division rule

In parallel circuits, the ratio of the current in one branch of a two-branch parallel circuit to the total current is equal to the ratio of the resistance of the other branch to the sum of the two resistances.

Problems on KVL and KCL

1. What is the voltage V_S across the open switch in the circuit shown in Fig. Q1?

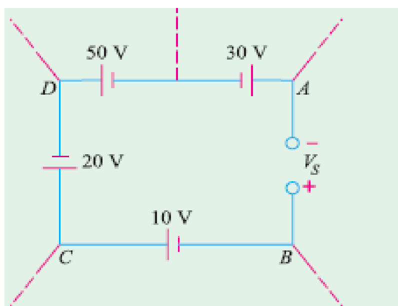


Fig. Q1

Solution:

We will apply KVL to find V_S. Starting from point A in the clockwise direction
$V_S + 10 - 20 - 50 + 30 = 0 \quad \therefore V_S = 30V$

2. Find the unknown voltage V_1 in the circuit of Fig. Q2.

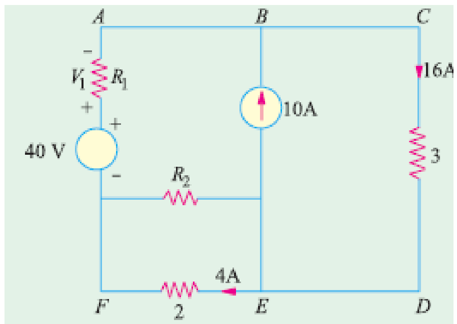


Fig. Q2

Solution:

Taking the outer closed loop ABCDEFA and applying KVL to it, we get
$(-16 \times 3) - (4 \times 2) + 40 - V_1 = 0$
$\therefore V_1 = -16V$

3.For the circuit shown in Fig. Q3, find V_{CE} and V_{AG}.

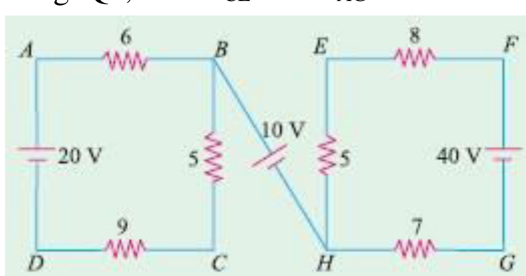


Fig. Q3

Solution:
Consider the two battery circuits of Fig Q3 separately. Current in the 20V battery circuit ABCD is $\frac{20}{6+5+9} = 1A$.
Similarly, current in the 40V battery circuit EFGH is $\frac{40}{5+8+7} = 2A$

For finding V_{CE} , we will find the algebraic sum of the voltage drops from point E to C via H and B.

$$\therefore V_{CE} = (-5 \times 2) + 10 - (5 \times 1) = -5V$$

The –ve sign shows that the point C is negative with respect to point E.

For finding V_{AG} , we will find the algebraic sum of the voltage drops from point E to C via H and B.

$$V_{AG} = (7 \times 2) + 10 + (6 \times 1) = 30V$$

4. Using Kirchhoff's Current Law and Ohm's law, find the magnitude and polarity of voltage V in Fig. Q4

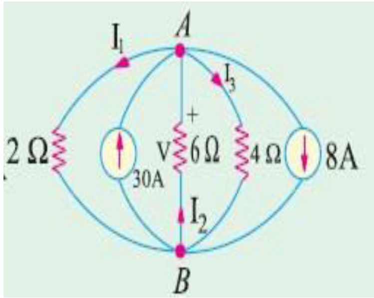


Fig. Q4

Solution:
Applying KCL to node A, we have $I_1-I_2+I_3=22$ ---(i)
Applying Ohm's law, we have
$I_1 = V/2$, $I_3= V/4$, $I_2= -V/6$

Substituting these values in eqn (i) ,we get V=24V
$I_1=12A, I_2=-4A, I_3=6A$
The negative sign of I2 indicates that actual direction of its flow is opposite to that of shown in Fig. Q4.

5.Determine the branch currents in the network of Fig. Q.5.

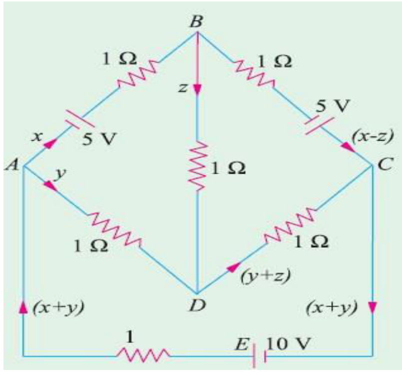


Fig. Q5

Solution:
Apply KCL to the closed circuit ABDA, we get
$5 - x - z + y = 0 \quad or \quad x - y + z = 5$

Similarly, circuit BCDB GIVES
$-(x - z) + 5 + (y + z) + z = 0$
or $x - y - 3z = 5$

From circuit ADCEA,we get
$-y - (y + z) + 10 - (x + y) = 0$
or $x + 3y + z = 10$

On solving we get $z = 0, x = 6.25A$ and $y = 1.24A$
Current in branch AB = current in branch $BC = 6.25A$
Current in branch $BD = 0$; current in branch AD = current in branch $DC = 1.25A$; current in branch $CEA = 7.5A$.

Self Assessment

1. Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistors of the circuit shown in Fig. Q.6. Also determine the potential difference across the external resistors.

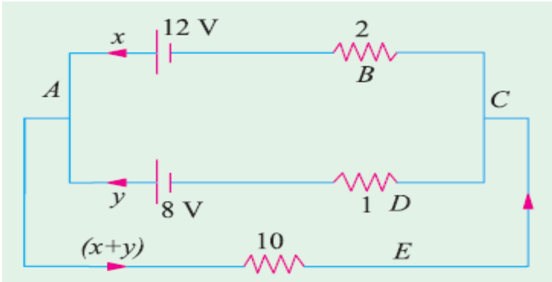


Fig. Q. 6

2. Find i_x in the circuit shown in Fig. Q.7

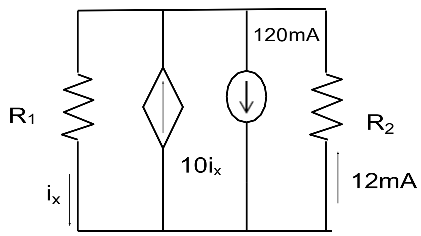
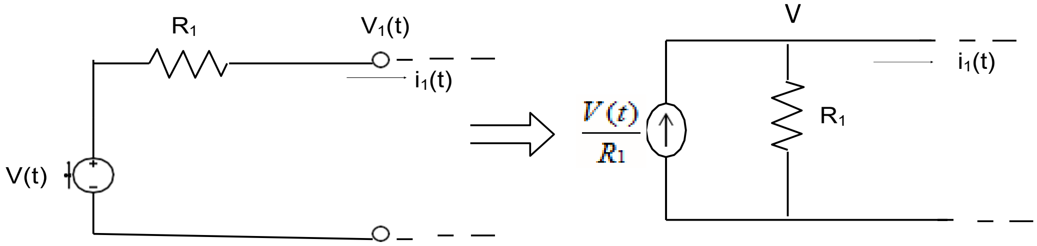


Fig. Q.7

Source Transformation

In network analysis it may be required to transform a practical voltage source into its equivalent practical current source and vice versa which are depicted in Fig.1.11. These are as explained follows.



Applying KVL,

$V(t) - i_1(t).R_1 - V_1(t) = 0$ or $i_1(t) = \frac{V(t)}{R_1} - \frac{V_1(t)}{R_1}$

Sources with equivalent terminal characteristics

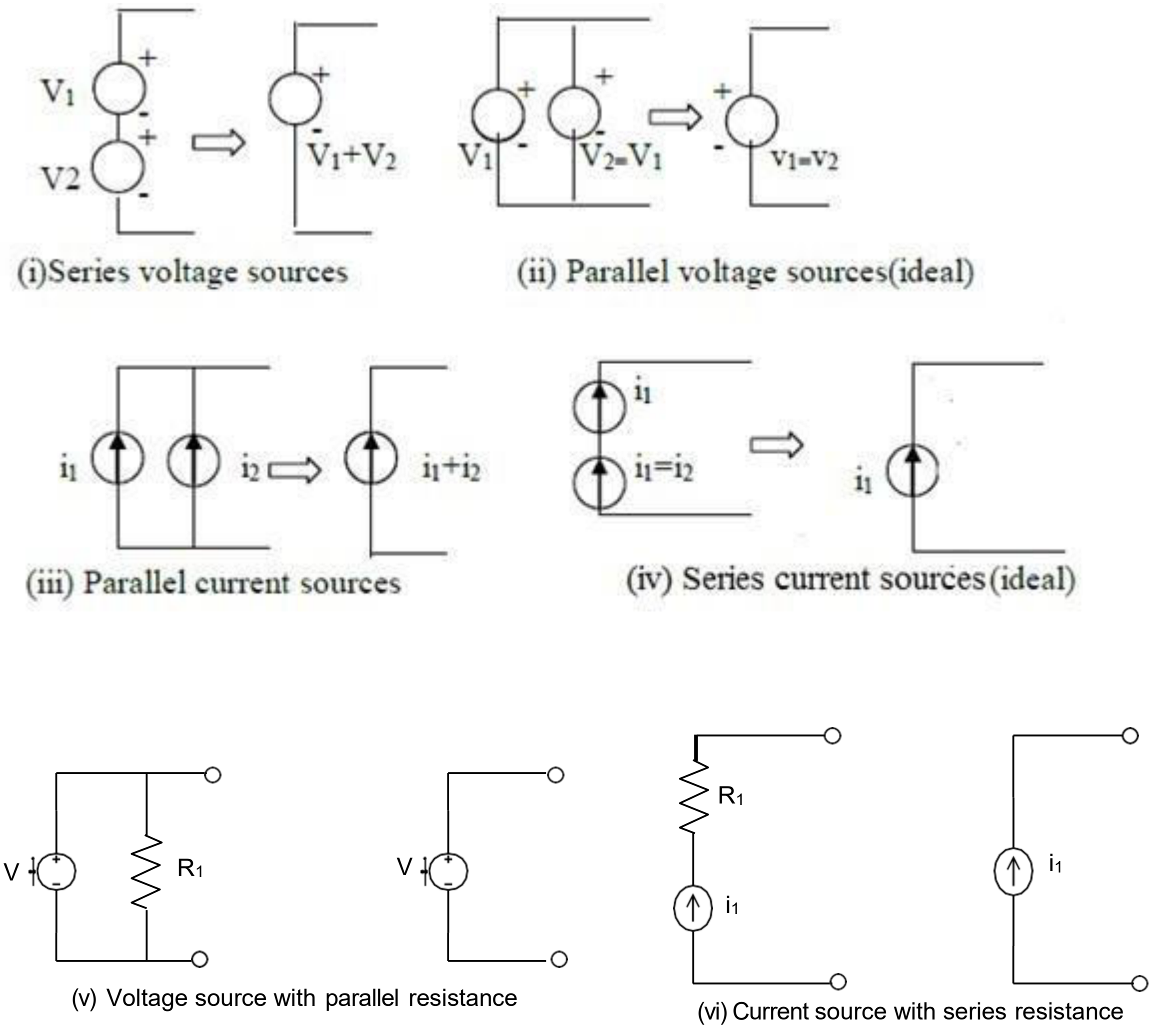


Fig.1.11: Source Transformation

Self Assessment

1.Using successive source transformation, simplify the network shown in Fig. Q8 betweenX &Y.

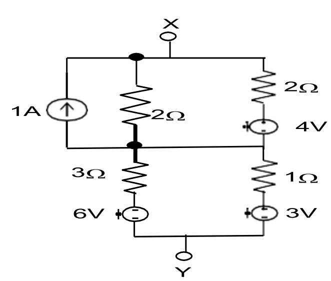


Fig.Q.8

Delta/Star Transformation

In solving networks by the applications of Kirchhoff's laws, one sometimes experiences great difficulty due to a large number of simultaneous equations that have to be solved. However such complicated network can be simplified by successively replacing delta meshes by equivalent star system and vice versa.

A delta connected network of three resistances (or impedances) R12, R23, and R31 can be transformed into a star connected network of three resistances (or impedances) R1, R2, and R3 as shown in Fig.1.12 using following transformations

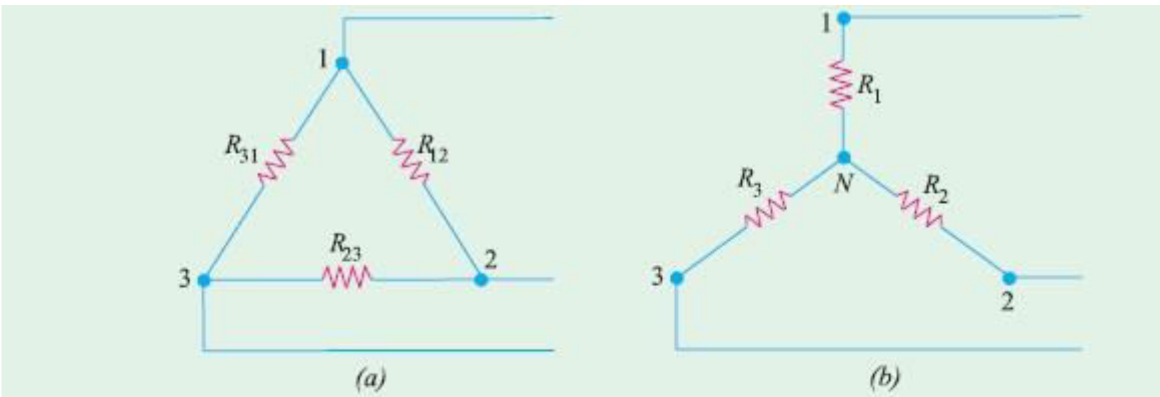


Fig.1.12: Source Transformation

$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} \;;\quad R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}} \; and \;\; R_3 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$$

Note: You can observe that in each of the above expressions, resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three resistances.

Star/Delta transformation

This transformation can be easily done by the following equations

$$R_{12} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_3} \;;\; R_{23} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_1} \; and \;\; R_{31} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2}$$

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistances.

Problems

1. Calculate the equivalent resistance between the terminals A and B in the network shown in Fig. Q.9.

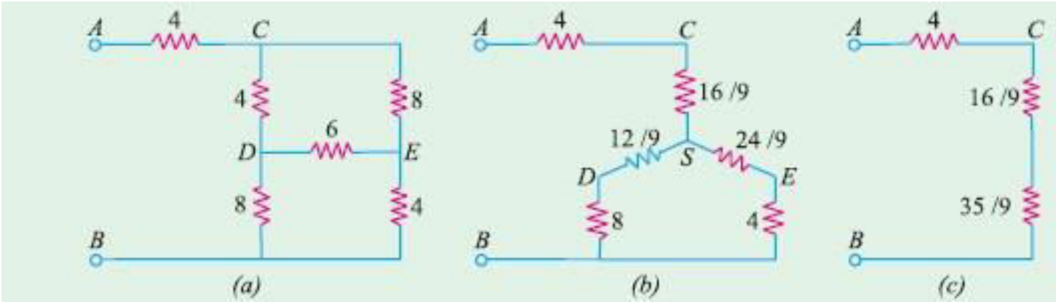


Fig. Q.9

Solution:
R_{CS}= 16/9Ω, R_{ES}=24/9Ω and R_{DS}=12/9Ω
R_{AB}= 4+(16/9)+(35/9)=87/9Ω

Self Assessment

1. Calculate the current flowing through the 10Ω resistor of Fig. Q.10.

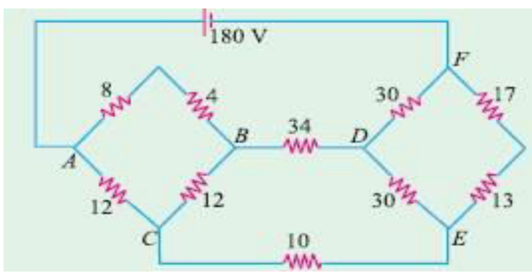


Fig. Q.10

2. A network of resistances is formed as shown in Fig.Q.11. Compute the network resistance measured between (i) A and B (ii) B and C (iii) C and A.

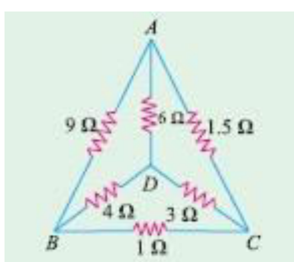


Fig. Q.11

Introduction to Nodal and Mesh Analysis

When we want to analyse a given network, we try to pick the minimum number of variables and the corresponding number of equations to keep the calculations to a minimum. Thus we would normally work with either currents only or voltages only. This can be achieved using these two analyses.

Mesh or Loop Analysis

Mesh Analysis involves solving electronic circuits via finding mesh or loop currents of the circuit. This is done by forming KVL equations for respected loops and solving the equations to find individual mesh currents. This method eliminates a great deal of tedious work involved in the branch current method.
We simply assume clockwise current flow in all the loops and find them to analyze the circuit. Also any independent current source in a loop becomes the loop current.

No. of loops= No. of branches - (No. of nodes-1)

Circuit with independent voltage sources

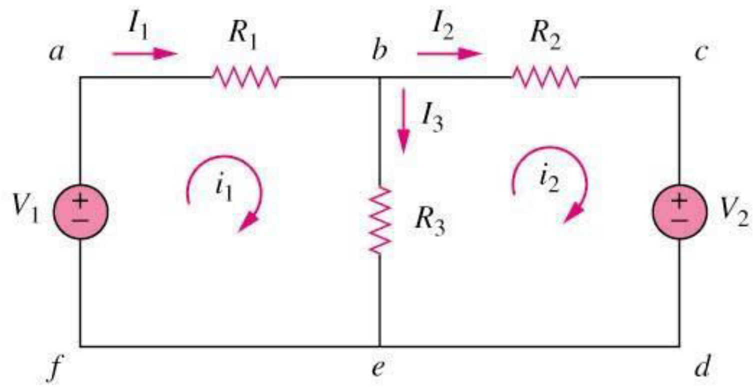


Fig.1.13: Mesh analysis for independent voltage sources

Using KVL for the circuit as shown in Fig.1.13, at loops 1 and 2, we form KVL equations using the current and components in the loops in terms of the loop currents. Important thing to look at it is the subtraction of the opposing loop current in the shared section of the loop.

Equations:

$R_1 \cdot i_1 + (i_1 - i_2) \cdot R_3 = V_1$

$R_2 \cdot i_2 + R_3 \cdot (i_2 - i_1) = -V_2$

i..e. $(R_1 + R_3) \cdot i_1 - i_2 \cdot R_3 = V_1$

$-R_3 \cdot i_1 + (R_2 + R_3) \cdot i_2 = -V_2$

Note: i_1 and i_2 are mesh current.
I_1, I_2 and I_3 are branch current.

$I_1 = i_1;\ I_2 = i_2;\ I_3 = (i_1 - i_2)$

Formalization: Network equations by inspection

$$\begin{pmatrix} (R_1 + R_3) & -R_3 \\ -R_3 & (R_2 + R_3) \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ -V_2 \end{pmatrix}$$

Impedance matrix   Mesh currents   Excitation

Use determinants and Cramer's rule for solving network equations through manipulation of their co-efficients.

Note:

Solving equations with two unknowns

Suppose the two given simultaneous equations are

$$ax + by = c$$
$$dx + ey = f$$

Here, the two unknown are x and y, a, b, d and e are coefficients of these unknowns whereas c and f are constants. The procedure for solving these equations by the method of determinants is as follows :

1. Write the two equations in the matrix form as $\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$
2. The *common* determinant is given as $\Delta = \begin{bmatrix} a & b \\ d & e \end{bmatrix} = ae - bd$
3. For finding the determinant for x, replace the co-efficients of x in the original matrix by the constants so that we get determinant Δ_1 given by $\Delta_1 = \begin{vmatrix} c & b \\ f & e \end{vmatrix} = (ce - bf)$
4. For finding the determinant for y, replace coefficients of y by the constants so that we get $\Delta_2 = \begin{vmatrix} a & c \\ d & f \end{vmatrix} = (af - cd)$
5. Apply Cramer's rule to get the value of x and y

$$x = \frac{\Delta_1}{\Delta} = \frac{ce - bf}{ae - bd} \quad \text{and} \quad y = \frac{\Delta_2}{\Delta} = \frac{af - cd}{ae - bd}$$

Solving equations with three unknowns

Let the three simultaneous equations be as under :

$$ax + by + cz = d$$
$$ex + fy + gz = h$$
$$jx + ky + lz = m$$

The above equations can be put in the matrix form as under :

$$\begin{bmatrix} a & b & c \\ e & f & g \\ j & k & l \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d \\ h \\ m \end{bmatrix}$$

The value of common determinant is given by

$$\Delta = \begin{vmatrix} a & b & c \\ e & f & g \\ j & k & l \end{vmatrix} = a\,(fl - gk) - e\,(bl - ck) + j(bg - cf)$$

The determinant for x can be found by replacing coefficients of x in the original matrix by the constants.

$$\therefore \qquad \Delta_1 = \begin{vmatrix} d & b & c \\ h & f & g \\ m & k & l \end{vmatrix} = d\,(fl - gk) - h\,(bl - ck) + m(bg - cf)$$

Similarly, determinant for y is given by replacing coefficients of y with the three constants.

$$\Delta_2 = \begin{vmatrix} a & d & c \\ e & h & g \\ j & m & l \end{vmatrix} = a\,(hl - mg) - e\,(dl - mc) + j\,(dg - hc)$$

In the same way, determinant for z is given by

$$\Delta_3 = \begin{vmatrix} a & b & d \\ e & f & h \\ j & k & m \end{vmatrix} = a\,(fm - hk) - e\,(bm - dk) + j\,(bh - df)$$

As per Cramer's rule $\qquad x = \dfrac{\Delta_1}{\Delta},\; y = \dfrac{\Delta_2}{\Delta},\; z = \dfrac{\Delta_3}{\Delta}$

Problems

1.Determine the current supplied by each battery in the circuit shown in Fig. Q.12.

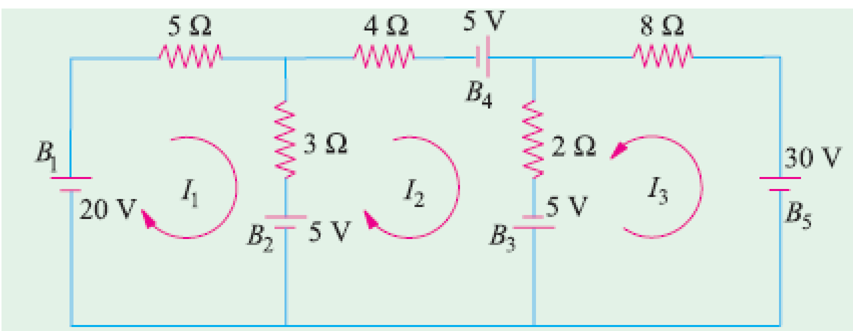


Fig. Q.12

Solution:

For loop 1 we get

$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$ or $8I_1 - 3I_2 = 15$

For loop 2 we have

$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$ or $3I_1 - 9I_2 + 2I_3 = -15$

Similarly, for loop 3, we get

$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$ or $2I_2 - 10I_3 = 35$

On solving, we get I_1 = 765/299 A, I_2 = 542/299 A and I_3 = -1875/598 A

So current supplied by each battery is

B_1=765/299 A $B_3 = I_2+I_3$ =2965/598 A B_5 = 1875/598 A

$B_2 = I_1-I_2$=220/299 A $B_4= I_2$=545/299 A

2. Use mesh analysis to compute the voltage $V_{10\Omega}$ in Fig. Q.13.

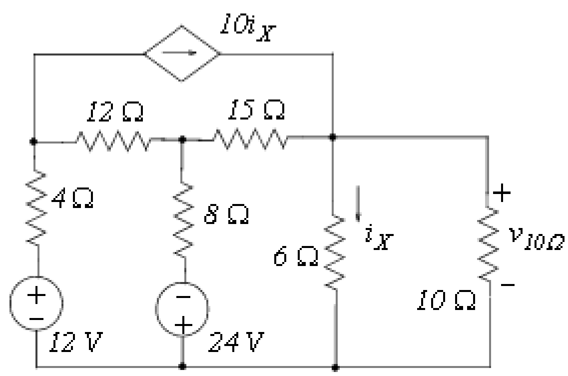


Fig. Q.13

Solution:

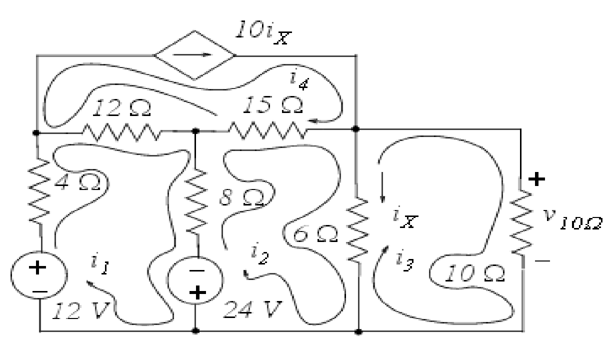


Fig. Q13.(a)

On applying KVL to Fig. Q13.(a) , We have

Mesh 1: $24i_1-8i_2-12i_4-24-12=0$ or $6i_1-2i_2-3i_4=9$

Mesh 2: $-8i_1+29i_2-6i_3-15i_4=-24$

Mesh 3: $-6i_2+16i_3=0$ or $-3i_2+8i_3=0$

Mesh 4: $i_4=10i_x=10(i_2-i_3)$ or $10i_2-10i_3-i_4=0$

On solving, we get

i_1=1.94A i_2=0.13A i_3=0.05A i_4=0.79A

Now, we find $V_{10\Omega}$ by ohm's law, that is,

$V_{10\Omega} = 10i_3 = 10 * 0.05 = 0.5V$

3. Using mesh analysis, find I_o for the circuit shown in Fig. Q14.

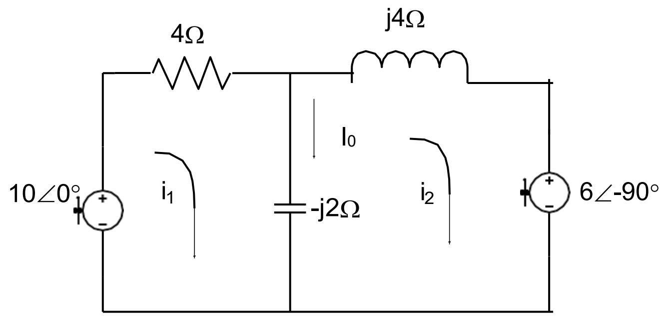


Fig. Q.14

Solution:

On applying KVL, we have

Mesh 1: $10\angle 0° - 4i_1 + j2(i_1 - i_2) = 0$ or $(2-j)i_1 + ji_2 = 5$

Mesh 2: $-j4i_2 + j2(i_2 - i_1) - 6/-90° = 0$ or $-j2i_1 + (-j4+j2)i_2 = 6\angle -90°$

$I_0 = (i_1 - i_2)$

On solving, we get

$i_1 = 2+j0.5$

$i_2 = 1-j0.5$

$I_0 = 1+j = 1.414/45°$

Nodal Analysis

The node-equation method is based directly on KCL. In nodal analysis, basically we work with a set of node voltages. It provides a general procedure for analyzing circuits using node voltages as the circuit variables.

For the application of this method, every junction in the network where three or more branches meet is regarded a node. One of these is regarded as the reference node or datum node or zero-potential node. Hence the number of simultaneous equations to be solved becomes (n-1) where n is the number of independent nodes. These node equations often become simplified if all voltage sources are converted into current sources.
Then we write the KCL equations for the nodes and solve them to find the respected nodal voltages. Once we have these nodal voltages, we can use them to further analyze the circuit.

Example

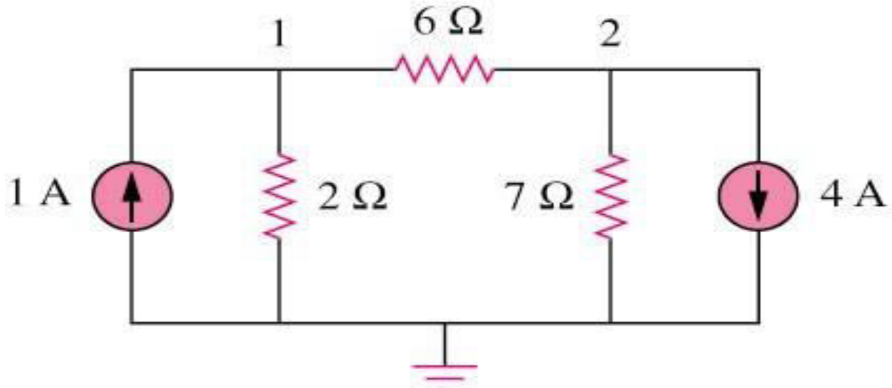


Fig.1.14: Nodal analysis for independent current sources

On applying KCL to the circuit shown in Fig.1.14, we get
At node 1
$1A = V_1/2 + (V_1-V_2)/6$ or $0.66V_1-0.166V_2 = 1A$

At node 2
$(V_1-V_2)/6 = V_2/7+4A$ or $0.166V_1 - 0.309V_2 = 4A$

On solving, we get
$V_1 = -2.01V$ and $V_2 = -14.02V$

Problems

1. Using nodal analysis, find the node voltages V_1 and V_2 in Fig. Q.15

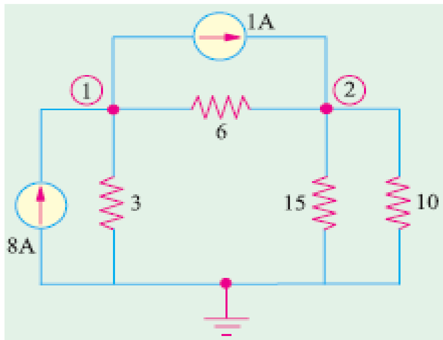


Fig. Q.15

Solution:

Applying KCL to node 1, we get
$8 - 1 - V_1/3 - (V_1-V_2)/6 = 0$ or $3V_1 - V_2 = 42$

Similarly, applying KCL to node 2, we get
$1 + (V_1-V_2)/6 - V_2/15 - V_2/10 = 0$ or $V_1 - 2V_2 = -6$

Solving for V_1 and V_2, we get
$V_1 = 18V$ and $V_2 = 12V$

2. Use nodal analysis to determine the value of current i in the network of Fig. Q.16

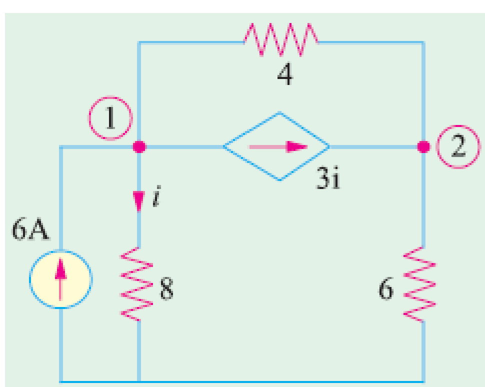


Fig. Q.16

Solution:

Applying KCL to node 1, we get

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3i$$

As seen, $i = \frac{V_1}{8}$. Hence, the above equation becomes

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3\frac{V_1}{8} \quad \text{or} \quad 3V_1 - V_2 = 24$$

Similarly, applying KCL to node 2, we get

$$\frac{V_1 - V_2}{4} + 3i = \frac{V_2}{6} \quad \text{or} \quad \frac{V_1 - V_2}{4} + 3\frac{V_1}{8} = \frac{V_2}{6} \quad \text{or } 3V_1 = 2V_2$$

From the above two equations, we get

$V_1 = 16V$ $\therefore i = 16/8 = 2A$

3. Find the value of the voltage v for the circuit of Fig. Q.17.

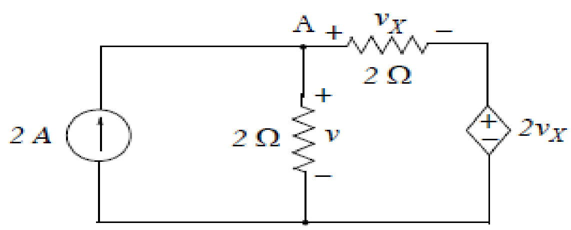


Fig. Q.17

Solution:

Application of KCL at Node A of the circuit below yields

$$\frac{v}{2} + \frac{v - 2v_x}{2} = 2 \text{ or } v - v_x = 2$$

Also by KVL

$v = v_x + 2v_x$

and by substitution

$v_x + 2v_x - v_x = 2 \text{ or } v_x = 1$

and thus $v = 3V$

Self Assessment:

1. Use mesh analysis to compute the current through the 6Ω resistor, and the power supplied (or absorbed) by the dependent source shown in Fig. Q18.

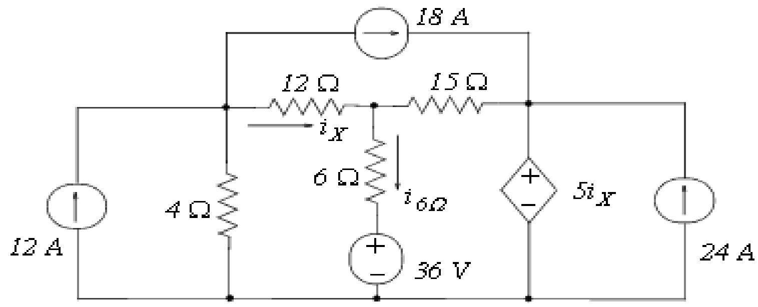


Fig. Q18

2. Use mesh analysis to find V_0 in the circuit of Fig. Q19.

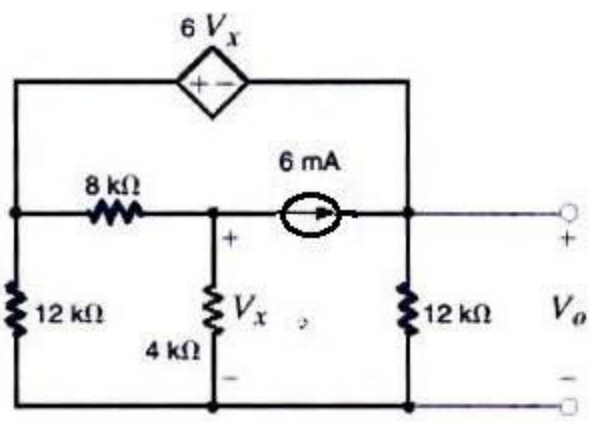


Fig. Q19

3. Use nodal analysis to compute the current through the 6Ω resistor and the power supplied (or absorbed) by the dependent source shown in Fig. Q20

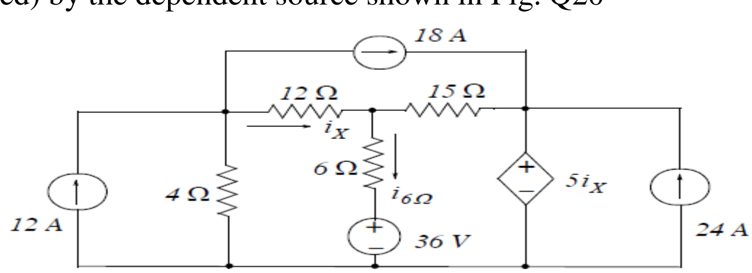
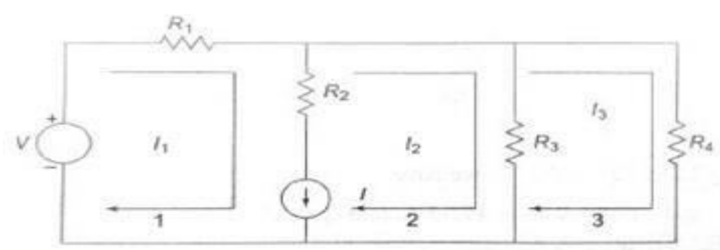


Fig.Q20

Super Mesh Analysis: If there is only current source between two meshes in the given network then it is difficult to apply the mesh analysis. Because the current source has to be converted into a voltage source in terms of the current source, write down the mesh equations and relate the mesh currents to the current source. But this is a difficult approach .This difficulty can be avoided by creating super mesh which encloses the two meshes that have common current source

Super Mesh: A super mesh is constituted by two adjacent meshes that have a common current source.

Let us illustrate this method with the following simple generalized circuit.



Solution:

Step (1): Identify the position of current source.

Here the current source is common to the two meshes 1 and 2. so, super mesh is
nothing but the combination of meshes 1 and 2 .
Step (2): Apply KVL to super mesh and to other meshes
Applying KVL to this super mesh (combination of meshes 1 and 2) we get

R1.I1 + R3 (I2 – I3) = V ....(1)

Applying KVL to mesh 3, we get

R3 (I3 – I2) + R4.I3 = 0 ....(2)

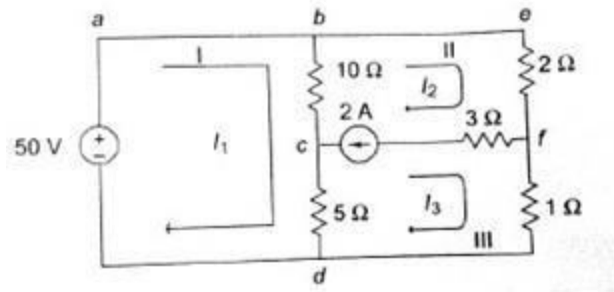
Step (3): Make the relation between mesh currents with current source to get third equation.

Third equation is nothing but he relation between I , I1 and I2 which is

I1 - I2 = I ....(3)

Step(4): Solve the above equations to get the mesh currents.

Example(1): Determine the current in the 5 Ω resistor shown in the figure below.



Solution:

Step(1): Here the current source exists between mesh(2) and mesh(3).Hence, super mesh is the combination of mesh(2) and mesh(3) .Applying KVL to the super mesh (combination of mesh 2 and mesh 3 after removing the branch with the current source of 2 A and resistance of 3 Ω) we get :

10(I2– I1) + 2.I2 + I3 + 5(I3 – I1) = 0

-15.I1 +12 I2 + 6.I3 = 0 ....(1)

Step (2): Applying KVL first to the normal mesh 1 we get :

10(I1 – I2) + 5(I1 – I3) = 50

15.I1 –10. I2 – 5.I3 = 50 ....(2)

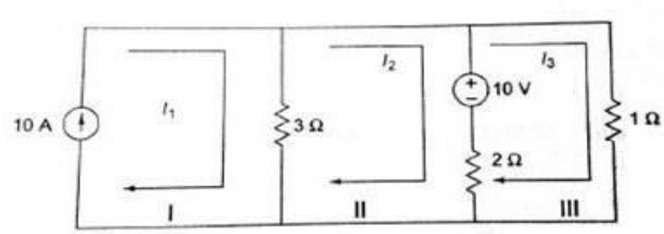
Step (3): We can get the third equation from the relation between the current source of 2 A , and currents I2 & I3 as :

I2 - I3 = 2 A ....(3)

Step (4): Solving the above three equations for I1, I2 and I3 we get I1 = 19.99 A I2 = 17.33 A and I3 = 15.33 A

The current in the 5 Ω resistance = I1 - I3 = 19.99 - 15.33 = 4.66 A

Example(2): Write down the mesh equations for the circuit shown in the figure below and find out the values of the currents I1, I2 and I3



Solution: In this circuit the current source is in the perimeter of the circuit and hence the first mesh is ignored. So, here no need to create the super mesh.

Applying KVL to mesh 1 we get :

$3(I2 - I1) + 2(I2 - I3) = -10$

$-3.I1 + 5.I2 - 2.I3 = -10$ ....(1)

Next applying KVL to mesh 2 we get :

$I3 + 2(I3 - I2) = 10$

$-2.I2 + 3.I3 = -10$.............. (2)

And from the first mesh we observe that....... I1 = 10 A .... (3)

And solving these three equations we get : I1 = 10 A, I2 = 7.27 A, I3 = 8.18 A

Nodal analysis:

Nodal analysis provides another general procedure for analyzing circuits nodal voltages as the circuit variables. It is preferably useful for the circuits that have many no. of nodes. It is applicable for the both planar and non planar circuits. This analysis is done by using KCL and Ohm's law.

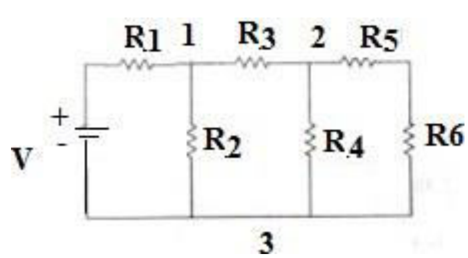
Node: It is a junction at which two or more branches are interconnected.

Simple Node: Node at which only two branches are interconnected.

Principal Node: Node at which more than two branches are interconnected.

Nodal analysis with example:

Determination of node voltages:

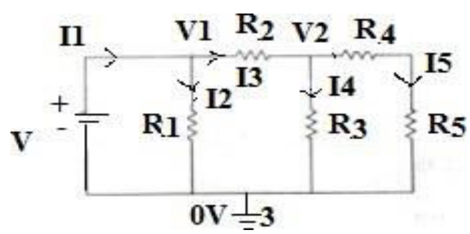


Procedure:

Step (1): Identify the no. nodes, simple nodes and principal nodes in the given circuit. Among all the nodes one node is taken as reference node. Generally bottom is taken as reference node. The potential at the reference node is 0v.

In the given circuit there are 3 principal nodes in which node (3) is the reference node.

Step (2): Assign node voltages to the all the principal nodes except reference node and assign branch currents to all branches.



Step (3): Apply KCL to those principal nodes for nodal equations and by using ohm's law express the node voltages in terms of branch current.

Applying KCL to node (1) ---- **1=I2+I3**

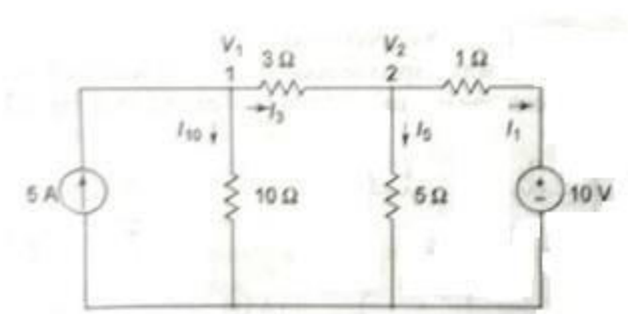
Using ohm's law, we get **(V-V1)/R1 =(V2-0)/R2 +(V1-V2)/R3. ....(1)**

Applying KCL to node (2) ---- **3=I4 +I5**

Using ohm's law, we get **(V1-V2)/R3 =(V4-0)/R4 +(V5-0)/R5 ....(2)**

Step(4): Solve the above nodal equations to get the node voltages.

Example: Write the node voltage equations and find out the currents in each branch of the circuit shown in the figure below.



Solution:

The node voltages and the directions of the branch currents are assigned as shown in given figure.
Applying KCL to node 1, we get: **5 = I10+ I3**

5= (V1-0)/10 +(V1-V2)/3

V1(13/30) -V2(1/3) = 5...........(1)

Applying KCL to node 2, we get: **I3= I5 + I1**

(V1-V2)/3 = (V2 -0)/5 + (V2-10) /1

V1(1/3)-V2(23/15) = -10 ....(2)

Solving the these two equations for V1 and V2 we get :

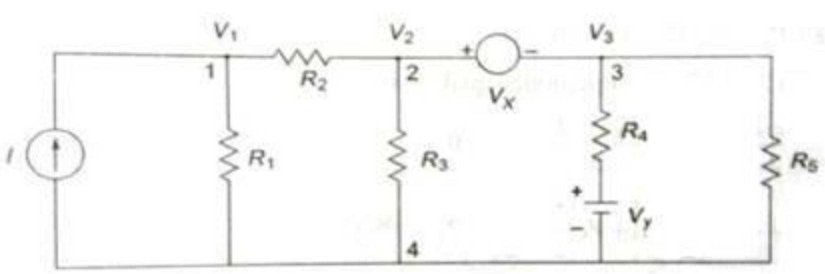
V1 = 19.85 V and V2 = 10.9 V and the currents are :

I10= V1/10 = 1.985A
I3 = (V1-V2)/3 = (19.85-10.9)/3 = 2.98A
I5 = V2/5 = 10.9/5 =2.18A
I1 = (V2-10) = (10.9-10)/1 = 0.9A

Super Node Analysis: If there is only voltage source between two nodes in the given network then it is difficult to apply the nodal analysis. Because the voltage source has to be converted into a current source in terms of the voltage source, write down the nodal equations and relate the node voltages to the voltage source. But this is a difficult approach .This difficulty can be avoided by creating super node which encloses the two nodes that have common voltage source.

Super Node: A super node is constituted by two adjacent nodes that have a common voltage source.

Example: Write the nodal equations by using super node analysis.



Procedure:

Step(1): Identify the position of voltage source.Here the voltage source is common to the two nodes 2 and 3.so, super node is nothing but the combination of nodes 2 and 3 .

Step (2): Apply KCL to super node and to other nodes.
Applying KCL to this super node (combination of meshes 2 and 3), we get

(V2-V1)/R2 + V2/R3 + (V3-Vy)/R4 + V3/R5 = 0 ....(1)

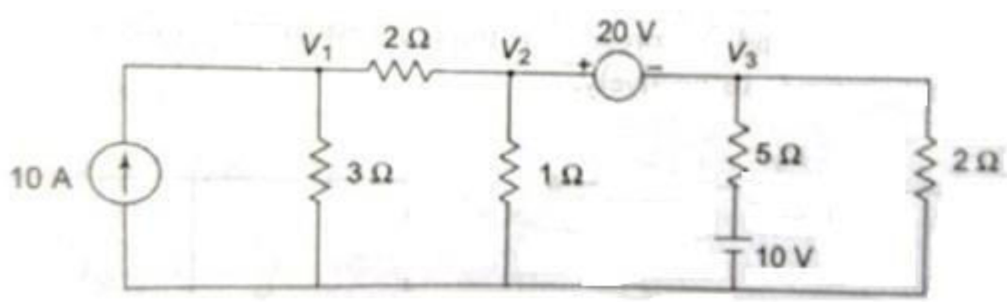
Applying KVL to node 1 ,we get

I = V1/R1 + (V1-V2)/R2.................(2)

Step (3): Make the relation between node voltages with voltage source to get third equation.

Third equation is nothing but the relation between VX , V2 and V3 which is
V2 - V3 = Vx ....(3)
Step (4): Solve the above nodal equations to get the node voltages.

Example: Determine the current in the 5 Ω resistor shown in the circuit below



Solution:

Applying KCL to node 1: 10 = V1/3 + (V1-V2)/2
V1 [1/3 + 1/2] - V2 /2 = 10

0.83 V1 - 0.5 V2 = 10 ....(1)

Next applying KCL to the super node2&3 :
(V2-V1)/2 + V2/1 + (V3-10)/5 + V3/2 = 0
-V1/2 + V2(1/2 + 1) V3(1/5 + 1/2) = 2
0.5 V1 + 1.5V2 + 0.7 V3 = 2 ....(2)
and the third and final equation is:
V2 - V3 = 20 ....(3)

Solving the above three equations we get V3 = -8.42 V

The current through the 5 Ω resistor I5 = [-8.42 -10] /5 = -3.68 A The negative sign indicates that the current flows towards the node 3.

Assignment Questions:

3. Calculate the current flowing through the 10Ω resistor of Fig. Q.9.

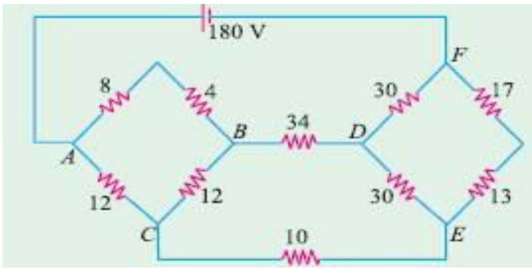


Fig. Q.9

2 Using Kirchhoff's Current Law and Ohm's law, find the magnitude and polarity of voltage V in Fig. Q10

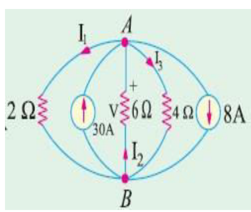


Fig. Q10

3.A network of resistances is formed as shown in Fig.Q.10. Compute the network resistance measured between (i) A and B (ii) B and C (iii) C and A.

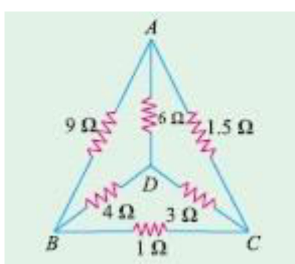


Fig. Q.10

6. Determine the current supplied by each battery in the circuit shown in Fig. Q.11.

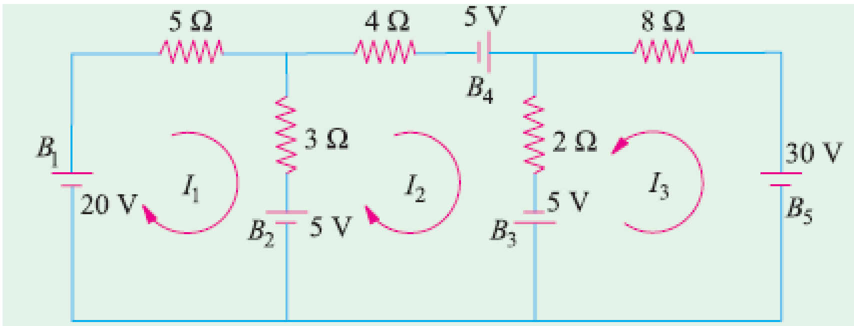


Fig. Q.11

5.Use nodal analysis to determine the value of current i in the network of Fig. Q.12

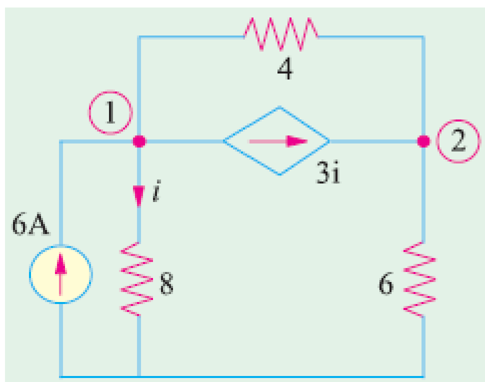


Fig. Q.12

Conclusion:

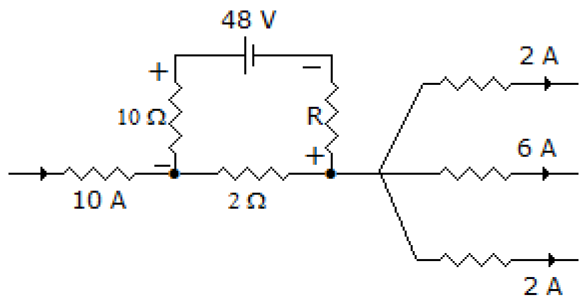
In this topic learner will be able to apply knowledge of KVL.KCL, Star to delta and Delta to star to solve numerical based on network simplification and it will be used to analyze the same.

Reference:

[1].Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi,2000
[2]. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education 2004
[3]. Paranjothi SR, "Electric Circuits Analysis," New Age International Ltd., New Delhi, 1996.

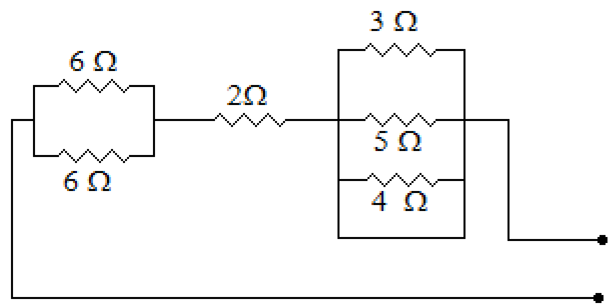
Post Test MCQs:

1. In figure the voltage drop across the 10 Ω resistance is 10 V. The resistance R



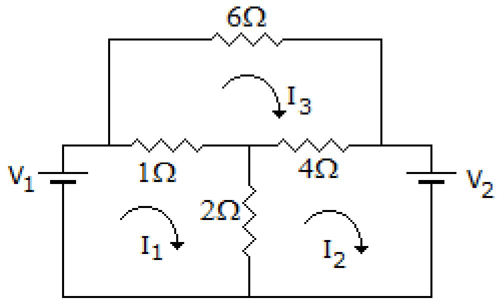
a. is 6 Ω
b. is 8 Ω
c. is 4 Ω
d. cannot be found

2. The resistance of the circuit shown is figure is



a. More than 6 Ω
b. 5 Ω
c. More than 4 Ω
d. Between 6 and 7 Ω

3. For the network in figure, the correct loop equation for loop 3 is

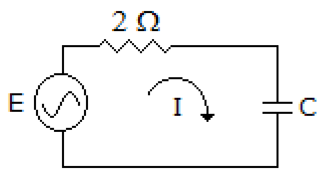


a. $-I_1 + 4I_2 + 11I_3 = 0$
b. $I_1 + 4I_2 + 11I_3 = 0$
c. $-I_1 - 4I_2 + 11I_3 = 0$
d. $I_1 - 4I_2 + 6I_3 = 0$

4. A Thermister is used for

   a. over voltage protection
   b. automatic light control
   c. temperature alarm circuit
   d. none of the above

5. In figure, E = 1 V (rms value). The average power is 250 mW. Then phase angle between E and I is



   a. 90°
   b. 60°
   c. 45°
   d. 30°

Unit-2 NETWORK THEOREM

AIM:

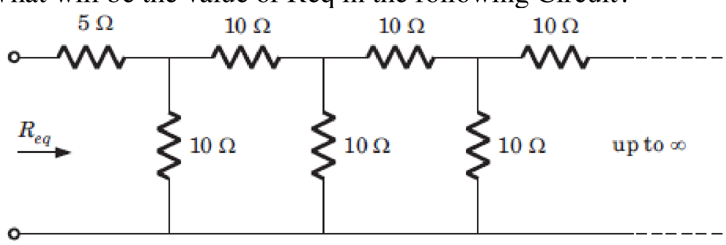
To learn techniques of solving circuits involving different active and passive elements.

Pre-Requisites:

Knowledge of Basic Mathematics – II & Basic Electronics Engineering

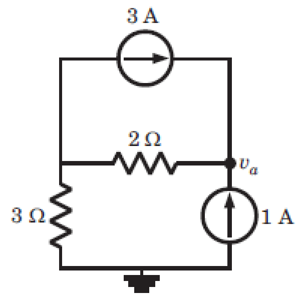
Pre - MCQs:

1. What will be the value of Req in the following Circuit?



a) 11.86 ohm
b) 10 ohm
c) 25 ohm
d). 11.18 ohm

2. What will be the value of Va in the following circuit?



a) -11 V
b). 11 V
c) 3 V
d) -3 V

3. What will be the value of Va in the following circuit?

a) 4.33 V
b) 4.09 V
c).8.67 V
d) 8.18 V

Network Theorems

Introduction:

Complex circuits could be analysed using Ohm's Law and Kirchoff's laws directly, but the calculations would be tedious. To handle the complexity, some theorems have been developed to simplify the analysis.

Superposition Theorem

The Superposition theorem states that in any linear bilateral network containing two or more independent sources (voltage or current sources or combination of voltage and current sources), the resultant current/voltage in any branch is the algebraic sum of currents/voltages caused by each independent sources acting alone, with all other independent sources being replaced meanwhile by their respective internal resistances as shown in Fig. 2.1 (a & b).

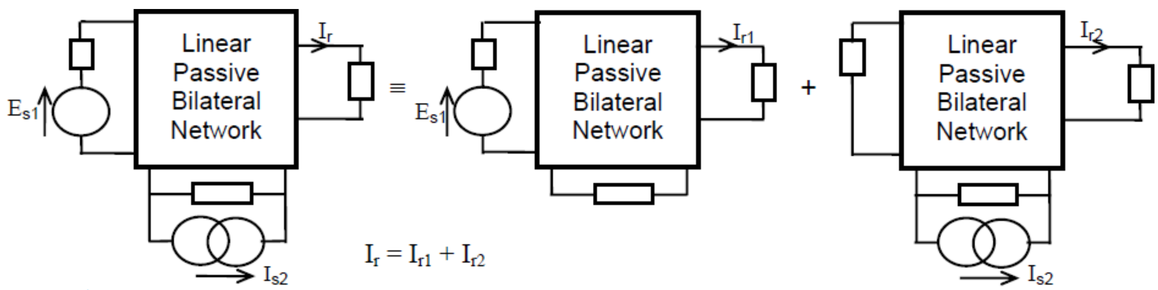


Fig. 2.1: (a) A Linear bilateral network (b) The resultant circuit

Procedure for using the superposition theorem

Step-1: Retain one source at a time in the circuit and replace all other sources with their internal resistances. i.e., Independent voltage sources are replaced by 0 V (short circuit) and Independent current sources are replaced by 0 A (open circuit).
Step-2: Determine the output (current or voltage) due to the single source acting alone using the mesh or nodal analysis.
Step-3: Repeat steps 1 and 2 for each of the other independent sources.
Step -4: Find the total contribution by adding algebraically all the contributions due to the independent sources.

Note: Dependent sources are left intact because they are controlled by circuit variables.

Example:

Use the superposition theorem to find **v** in the circuit shown in Fig. 2.2.

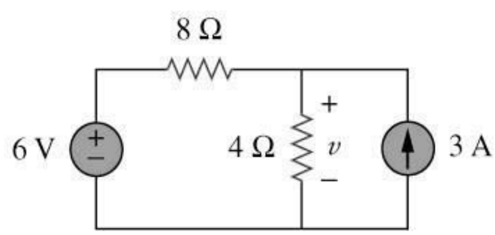


Fig. 2.2

Solution: Consider voltage source only as shown in Fig. 2.2(a) (current source 3A is discarded by open circuit)

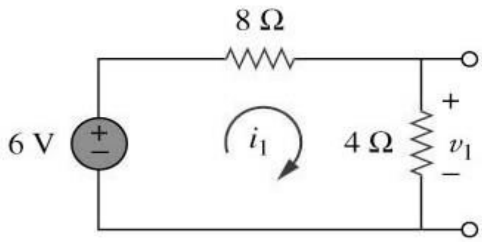


Fig. 2.2(a)

$$v_1 = 4i_1 = 4 \times (6/12) = 2V$$

Consider current source only as shown in Fig. 2.2(b) (voltage source 6V is discarded by short circuit)

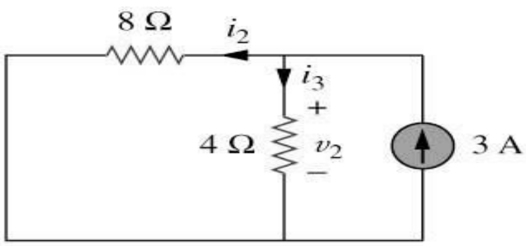


Fig. 2.2(b)

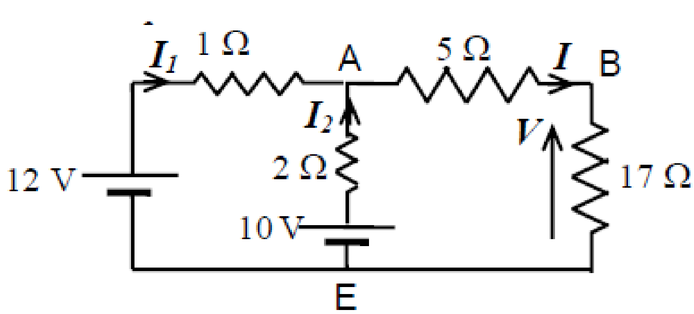
$$v_2 = 4i_3 = 4 \times (3 \times 8/12) = 8V$$

Total voltage $v = v_1 + v_2 = 10$V

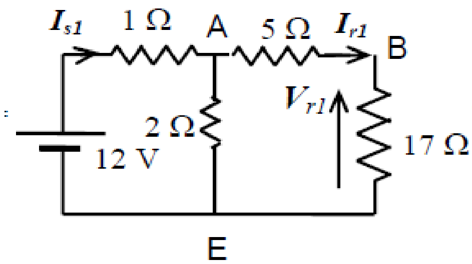
Problems

1. Use the superposition theorem to find **I** in the circuit shown in Fig. 26

Fig. 26



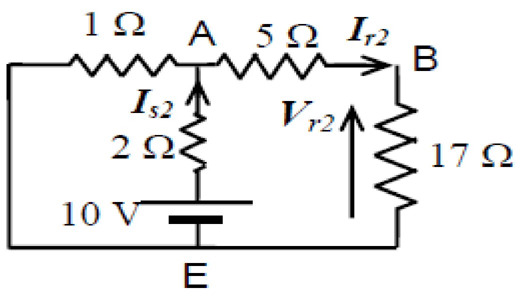
Solution:
Consider only 12V source



$I_{s1}= \frac{12}{1+2\,//\,(5+17)} = 4.2359\,A$

$\therefore\ I_{r1}= \frac{2}{2+(5+17)}$ x $4.2359 = 0.3529\,A$

Consider only 10V source



$I_{s2}= \frac{10}{2+1\,//\,(5+17)} = 3.382A$

$\therefore\ I_{r2}= \frac{1}{1+(5+17)}$ x $3.38235 = 0.14706\,A$

From superposition theorem, $I= I_{r1}+I_{r2} = 0.5$A

2. Use the superposition theorem to find **I** in the circuit shown in Fig. 27

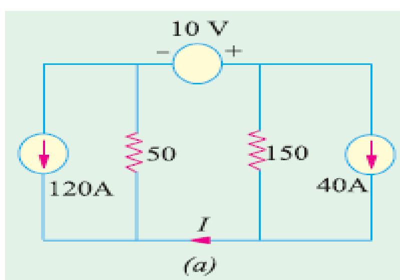
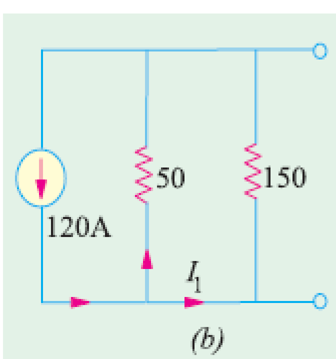


Fig. 27

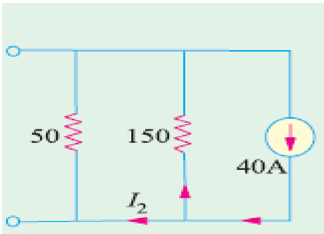
Solution:

Consider only 120A source



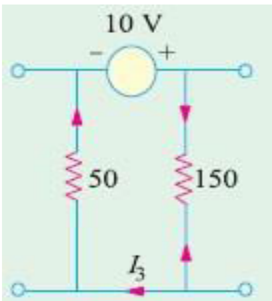
Using the current divider rule, we get
I_1=120 x 50/200= 30 A

Consider only 40A source
I_2=40 x 150/200= 30 A



(c)

Consider only 10V source



(d)

Using Ohm's law I_3 = 10/200 = 0.05 A

Using superposition theorem, Since I_1 and I_2 cancel out, $I=I_3$=0.05 A

3. Find the current *i* using superposition theorem for the circuit shown in Fig.28

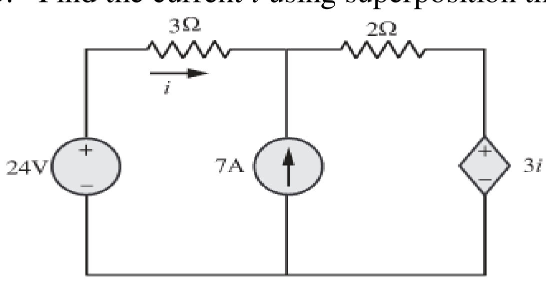


Fig. 28

Solution:

As a first step in the analysis, we will find the current resulting from the independent voltage source. The current source is deactivated and we have the circuit as shown in Fig. 28(a)

Applying KVL clockwise around loop shown in Fig. 3.12, we find that

$$5i_1+3i_1-24=0$$
$$i_1= 3\ A$$

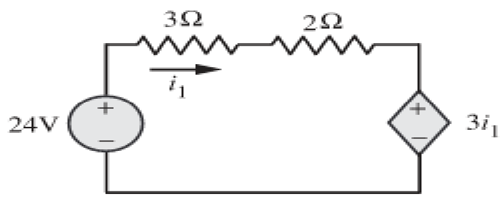


Fig. 28(a)

As a second step, we set the voltage source to zero and determine the current i_2 due to the current source as shown in Fig.28(b).

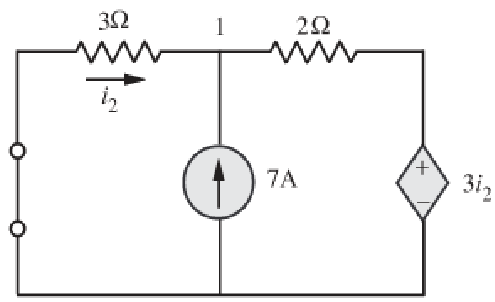


Fig. 28(b)

Applying KCL at node 1, we get

$$i_2 + 7 = \frac{v_1 - 3i_2}{2}$$

and $i_2 = \frac{0-v_1}{3}$

we get , $v_1 = -3i_2$

On substituting for v_1 , we get $i_2 = -\frac{7}{4}A$

Thus, the total current $i = i_1 + i_2 = \frac{5}{4}A$

4. For the circuit shown in Fig.29, find the terminal voltage V_{ab} using superposition principle.

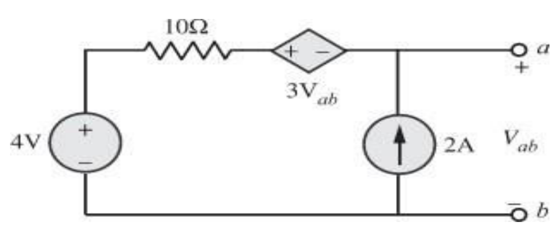
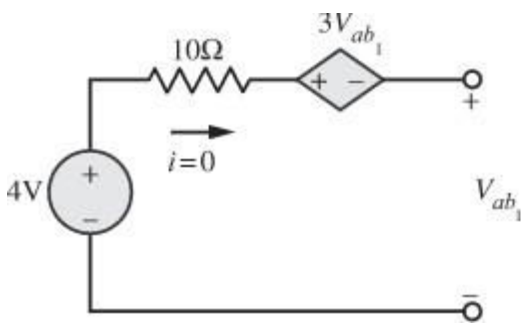


Fig.29

Solution:

Consider 4V source

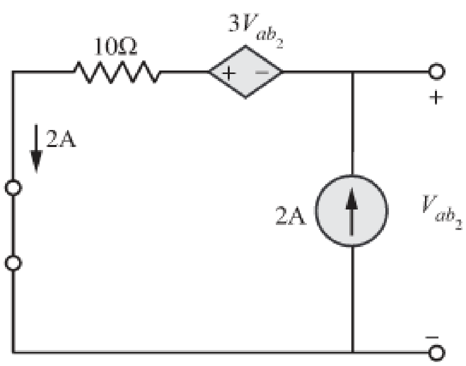


Apply KVL, we get

$4 - 10 \times 0 - 3V_{ab1} - V_{ab1} = 0$

$V_{ab1} = 1V$

Consider 2A source



Apply KVL, we get $-10 \times 2 + 3V_{ab2} + V_{ab2} = 0$

$V_{ab2} = 5V$

According to superposition principle, $V_{ab} = V_{ab1} + V_{ab2} = 6V$

Self Assessment

1. Find the current flowing in the branch XY of the circuit shown in Fig.30 by superposition theorem.

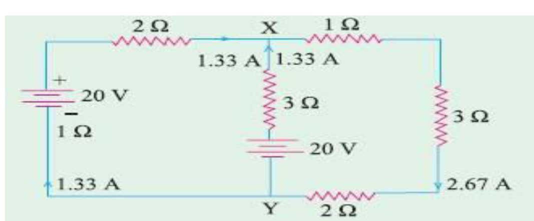


Fig.30

(Ans: 1.33 A)

2. Apply Superposition theorem to the circuit of Fig.31 for finding the voltage drop V across the 5Ω resistor.

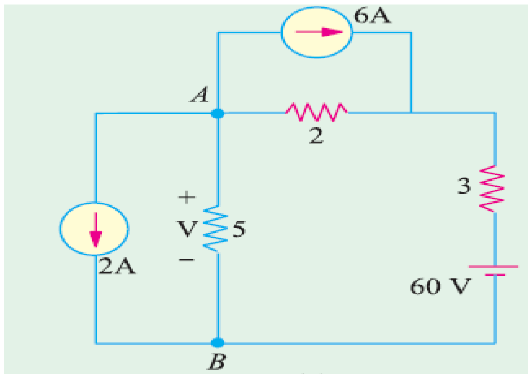


Fig. 31 (Ans: 19 V)

3. Find the voltage V_1 for the circuit shown in Fig. 32 using the superposition principle.

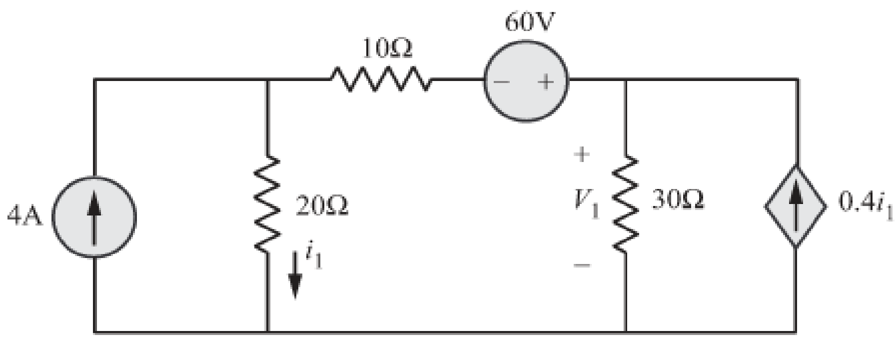


Fig. Q32 (Ans: 82.5 V)

4. Find I for the circuit shown in Fig. 33 using the superposition theorem.

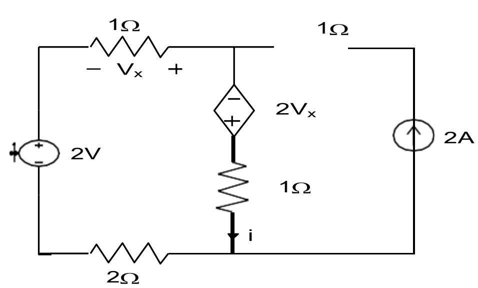


Fig.33

(Ans: 2 A)

Remarks: Superposition theorem is most often used when it is necessary to determine the individual contribution of each source to a particular response.

Limitations: Superposition principle applies only to the current and voltage in a linear circuit but it cannot be used to determine power because power is a non-linear function.

Thevenin's theorem

In section 2.1, we saw that the analysis of a circuit may be greatly reduced by the use of superposition principle. The main objective of Thevenin's theorem is to reduce some portion of a circuit to an equivalent source and a single element. This reduced equivalent circuit connected to the remaining part of the circuit will allow us to find the desired current or voltage. Thevenin's theorem is based on circuit equivalence.

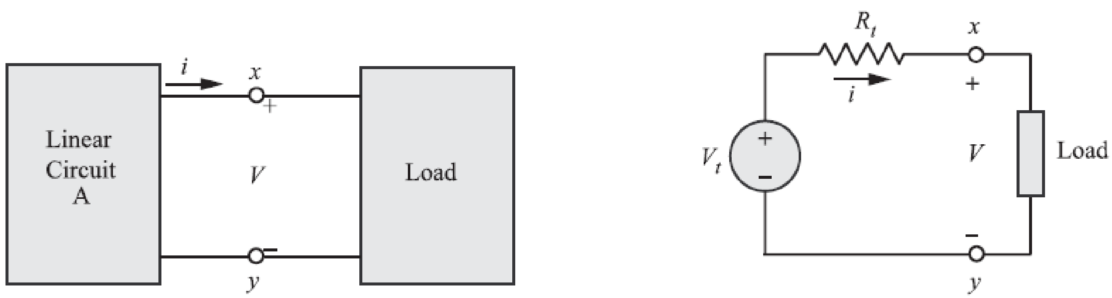


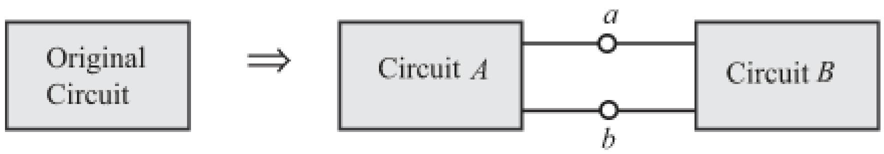
Fig.2.3: (a) A Linear two terminal network (b) The Thevenin's equivalent circuit

The Thevenin's theorem may be stated as follows:

A linear two–terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_t in series with a resistor R_t, Where V_t is the open–circuit voltage at the terminals and R_t is the input or equivalent resistance at the terminals when the independent sources are turned off or R_t is the ratio of open–circuit voltage to the short–circuit current at the terminal pair which is as shown in Fig. 2.3(a & b).

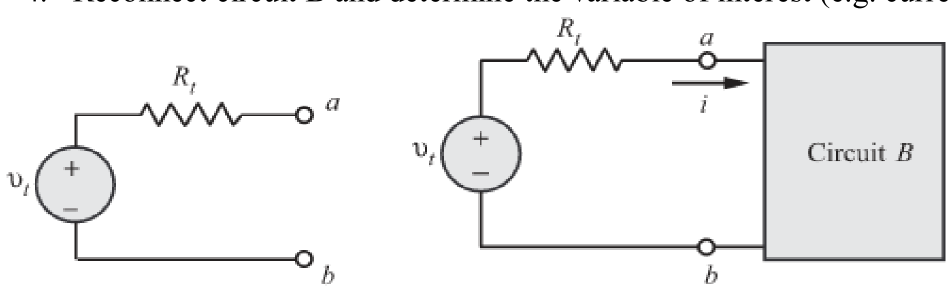
2.2.1 Action plan for using Thevenin's theorem :

1. Divide the original circuit into circuit A and circuit B



In general, circuit B is the load which may be linear or non-linear. Circuit A is the balance of the original network exclusive of load and must be linear. In general, circuit may contain independent sources, dependent sources and resistors or other linear elements.

2. Separate the circuit from circuit B
3. Replace circuit A with its Thevenin's equivalent.
4. Reconnect circuit B and determine the variable of interest (e.g. current 'i' or voltage 'v')



Procedure for finding R_t

Three different types of circuits may be encountered in determining the resistance, R_t

(i) If the circuit contains only independent sources and resistors, deactivate the sources and find R_t by circuit reduction technique. Independent current sources, are deactivated by opening them while independent voltage sources are deactivated by shorting them.

(ii) If the circuit contains resistors, dependent and independent sources, follow the instructions described below:

(a) Determine the open circuit voltage v_{oc} with the sources activated.

(b) Find the short circuit current i_{sc} when a short circuit is applied to the terminals a-b

(c) $R_t = \frac{v_{oc}}{i_{sc}}$

(iii) If the circuit contains resistors and only dependent sources, then

(a) $v_{oc} = 0$ (since there is no energy source)

(b) Connect 1A current source to terminals a-b and determine v_{ab}

(c) $R_t = \frac{v_{ab}}{1}$

For all the cases discussed above, the Thevenin's equivalent circuit is as shown in Fig. 2.4.

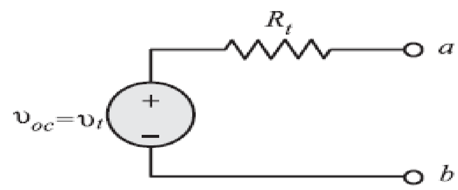


Fig. 2.4: The Thevenin's equivalent circuit

Problems

1. Using the Thevenin's theorem, find the current *i* through $R = 2\Omega$ for the circuit shown in Fig. 34.

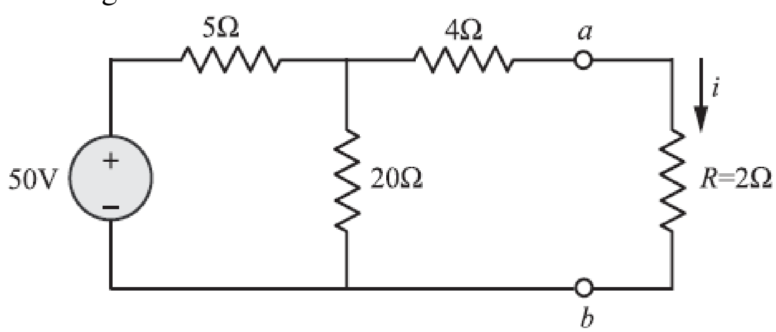
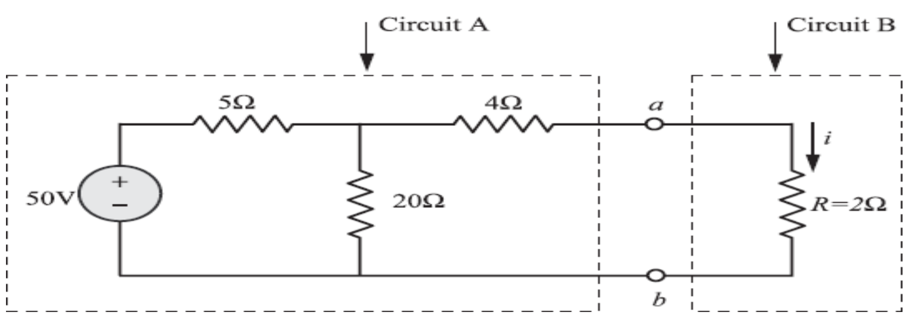


Fig. 34

Solution:



Since we are interested in the current i through R, the resistor R is identified as circuit B and the remainder as circuit A. After removing the circuit B, circuit A is as shown in Fig.34(a).

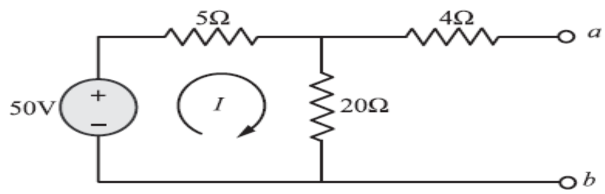


Fig.34(a)

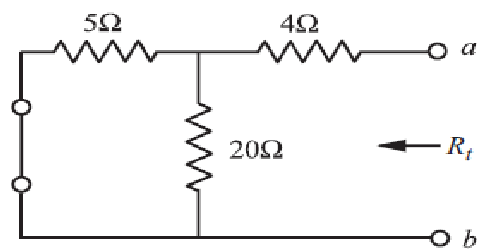
Referring to Fig. 34(a)

$$-50 + 25I = 0$$

$$I = 2A$$

Hence $V_{ab} = V_{oc} = 20(I) = 40V$

To find R_t, we have to deactivate the independent voltage source. Accordingly, we get the circuit in Fig.34(b).



$$R_t = (5\Omega || 20\,\Omega) + 4\,\Omega = 8\,\Omega$$

Fig.34(b)

Thus, we get the Thevenin's equivalent circuit which is as shown in Fig.34(c)

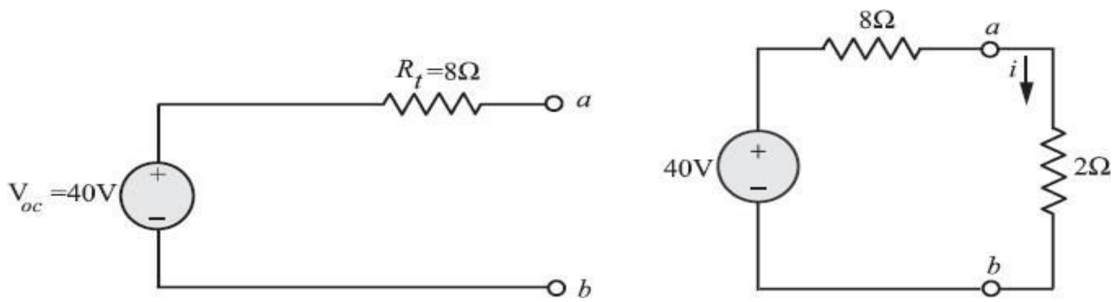


Fig.34(c).

Reconnecting the circuit B to the Thevenin's equivalent circuit as shown in Fig.34(c), we get

$i = 40/10 = 4A$

2. Find V_0 in the circuit of Fig.35 using Thevenin's theorem.

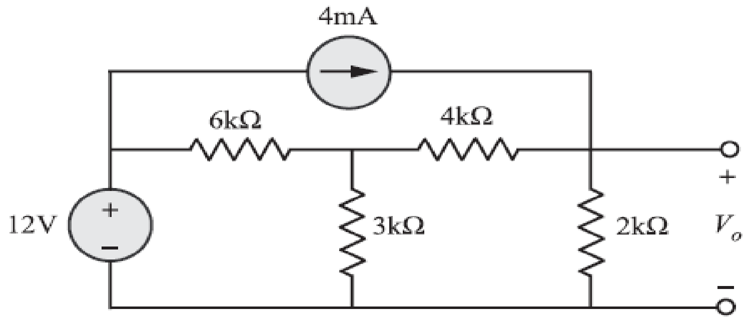


Fig.35

Solution:

To find V_{oc} :

Since we are interested in the voltage across 2 kΩ resistor, it is removed from the circuit of Fig.35 and so the circuit becomes as shown in Fig.35(a)

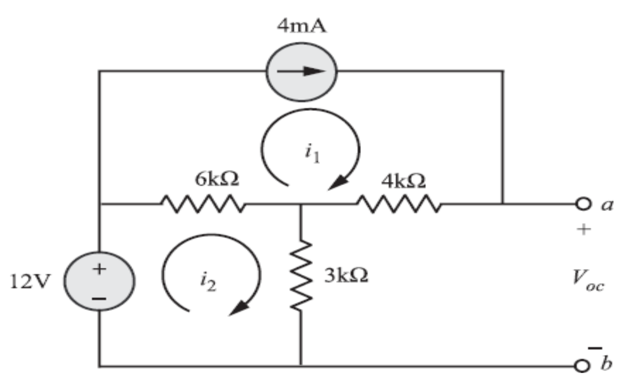


Fig.35(a)

By inspection, $i_1 = 4mA$

Applying KVL to mesh 2, we get

$12 - 6k \times (i_2 - i_1) - 3k \times i_2 = 0$

Solving, we get $i_2 = 4mA$

Applying KVL to the path 4 kΩ -> a-b -> 3 kΩ, we get

$V_{oc} - 4k \times i_1 - 3k \times i_2 = 0$

On solving, $V_{oc} = 28V$

To find R_t :

Deactivating all the independent sources, we get the circuit diagram shown in Fig.35(b)

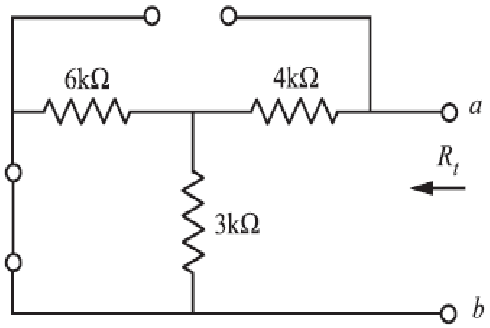


Fig.35(b)

$$R_t = R_{ab} = 4k\Omega + (6k\Omega||3k\Omega) = 6k\Omega$$

Hence, the Thevenin equivalent circuit is as shown in Fig.35(c).

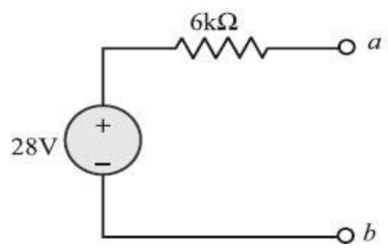


Fig.35(c)

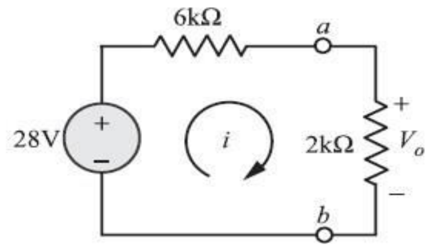


Fig.35(d)

If we connect the 2kΩ resistor to this equivalent network, we obtain the circuit of Fig.35(d).

$$V_0 = 2k \times i = 7V$$

3. Find the Theveni's equivalent for the circuit shown in Fig.36 with respect to terminals a-b.

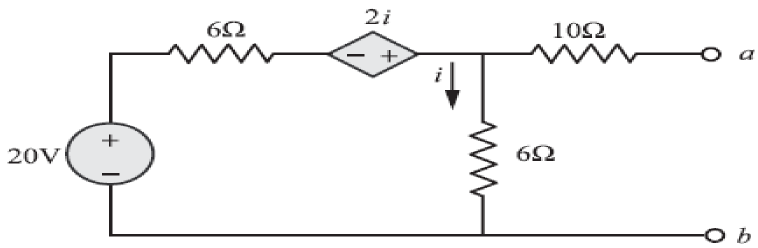


Fig.36

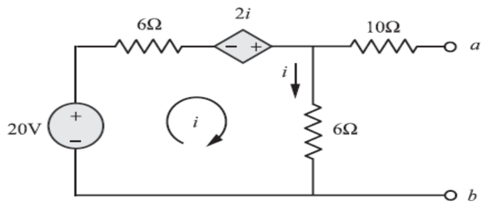


Fig.36(a)

Solution:

To find $V_{oc} = V_{ab}$:

Applying KVL around the mesh of Fig.36(a), we get

$20 - 6i + 2i - 6i = 0$

On solving, $i = 2A$

Since there is no current flowing in 10Ω resistor,

$V_{oc} = 6i = 12V$

To find R_t :

Since both dependent and independent sources are present, Thevenin's resistance is found using the relation, $R_t = \frac{v_{oc}}{i_{sc}}$

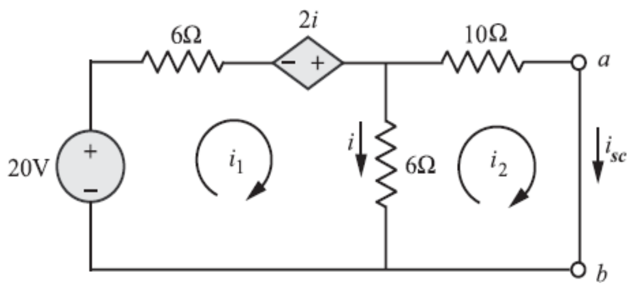


Fig.36(b)

Applying KVL clockwise for mesh 1 of Fig.36(b), we get

$20 - 6i_1 + 2i - 6(i_1 - i_2) = 0$

Since $i = i_1 - i_2$

Above equation becomes $10i_1 - 4i_2 = 20$

Applying KVL clockwise for mesh 2, we get

$10i_2 + 6(i_2 - i_1) = 0$

Solving the above two mesh equations, we get

$$i_{sc} = i_2 = \frac{120}{136} A$$

Hence $R_t = \frac{v_{oc}}{i_{sc}} = 13.6\Omega$

Self Assessment

1. For to the circuit shown in Fig.37, find the Thevenin's equivalent circuit at the terminals a-b.

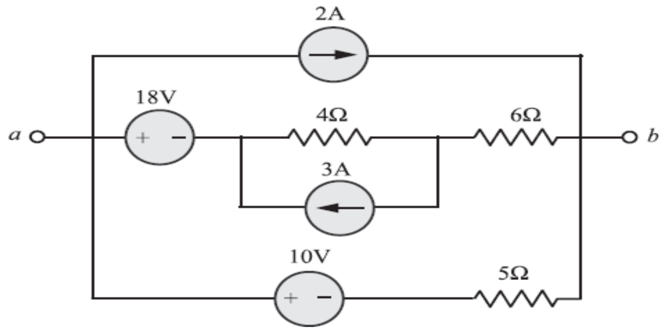


Fig.37

(Ans: V_{oc} = 10V, R_t = 3.33Ω)

2. For the circuit shown in Fig.38, find the Thevenin's equivalent circuit between terminals a and b.

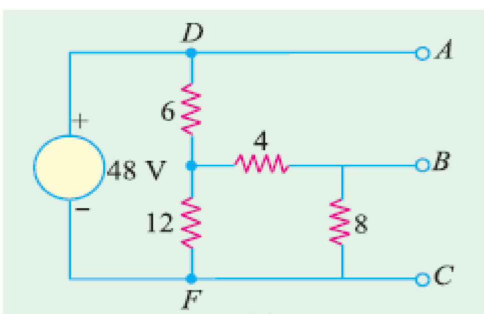


Fig.38 (Ans: V_{oc} = 32V, R_t =4Ω)

3. For the circuit shown in Fig.39, find the Thevenin's equivalent circuit between terminals a and b.

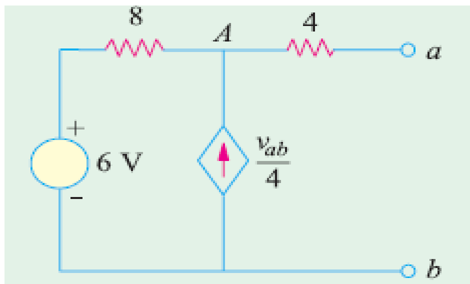


Fig.39 (Ans: V_{oc} = 12V, R_t =0.5Ω)

4. Find the Thevenin's equivalent circuit as seen from the terminals a-b . Refer the circuit diagram shown in Fig.40

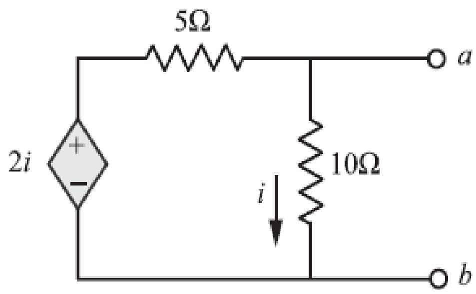


Fig.40

Hint: Since the circuit has no independent sources, i = 0 when the terminals a-b are open. Therefore V_{oc} = 0. Hence, we choose to connect a source of 1 A at the terminals a-b then, after finding V_{ab}, the Thevenin resistance is, $R_t = \frac{v_{ab}}{1}$

(Ans: V_{oc} = 0 ; R_t = 3.8Ω)

Norton's theorem

Norton's theorem is the dual theorem of Thevenin's theorem where the voltage source is replaced by a current source.
Norton's theorem states that a linear two-terminal network shown in Fig. 2.5(a) can be replaced by an equivalent circuit consisting of a current source i_N in parallel with resistor R_N, where i_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off. If one does not wish to turn off the independent sources, then R_N is the ratio of open circuit voltage to short–circuit current at the terminal pair.

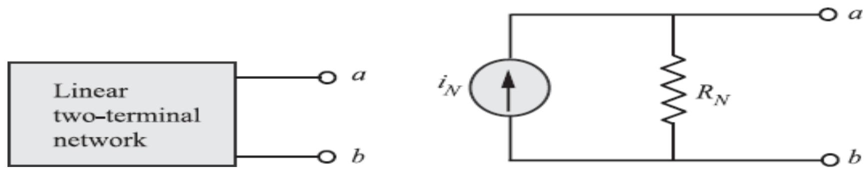


Fig.2.5: (a) A Linear two terminal network (b) The Norton's equivalent circuit

Fig. 2.5(b) shows Norton's equivalent circuit as seen from the terminals a-b of the original circuit shown in Fig. (a). Since this is the dual of the Thevenin's circuit, it is clear that $R_N = R_t$ and $i_N = \frac{v_{oc}}{R_t}$. In fact, source transformation of Thevenin's equivalent circuit leads to Norton's equivalent circuit.

Procedure for finding Norton's equivalent circuit:

(1) If the network contains resistors and independent sources, follow the instructions below:
(a) Deactivate the sources and find R_N by circuit reduction techniques.
(b) Find i_N with sources activated.

(2) If the network contains resistors, independent and dependent sources, follow the steps given below:
(a) Determine the short-circuit current i_N with all sources activated.
(b) Find the open-circuit voltage v_{oc}.
(c) $R_t = R_N = \frac{v_{oc}}{i_N}$

(3) If the network contains only resistors and dependent sources, follow the procedure described below:
(a) Note that $i_N = 0$.
(b) Connect 1A current source to the terminals a-b and find v_{ab}.
(c) $R_t = \frac{v_{ab}}{1}$

Note: Also, since $v_t = v_{oc}$ and $i_N = i_{sc}$

$$R_t = \frac{v_{oc}}{i_{sc}} = R_N$$

Problems

1. Find the Norton equivalent for the circuit of Fig. 41

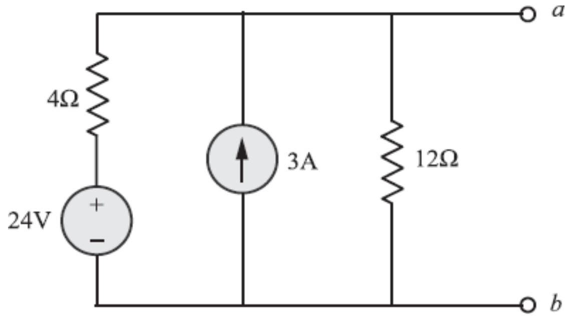


Fig. 41

Solution:

As a first step, short the terminals a-b. This results in a circuit as shown in Fig. 41(a)

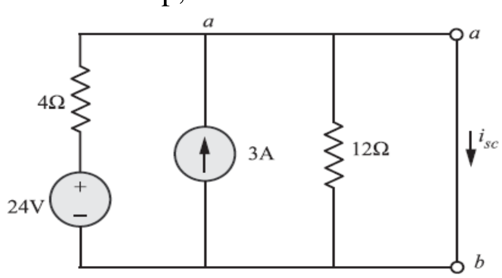


Fig. 41(a)

Applying KCL at node a, we get

$$\frac{(24-0)}{4} + 3 = i_{sc}$$

So $i_{sc} = 9$

To find R_N, deactivate all the independent sources, resulting in a circuit diagram as shown in Fig. Q41(b). We find R_N in the same way as R_t in the Thevenin's equivalent circuit.

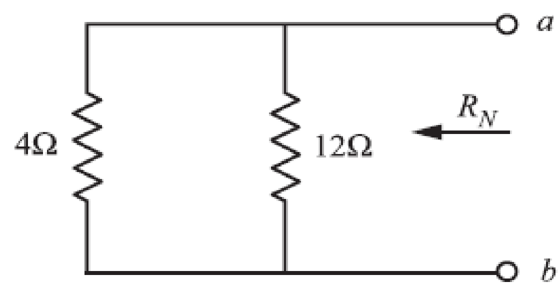


Fig. 41 (b)

$$R_N = \frac{48}{16} = 3\Omega$$

Thus, we obtain Norton equivalent circuit as shown in Fig. Q41(c)

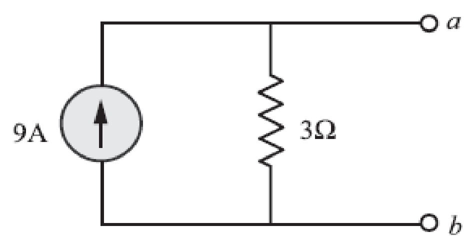


Fig. 41(c)

2. Find i_0 in the network of Fig.42 using Norton's theorem.

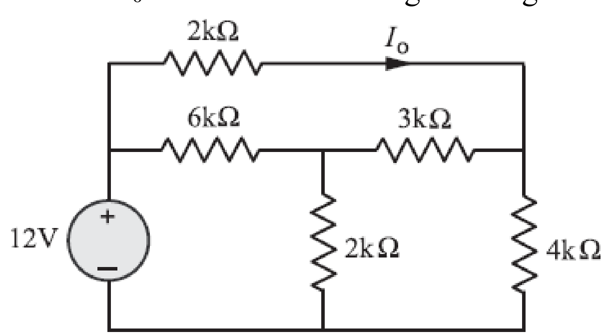


Fig. 42

Solution:

We are interested in i_0, hence the 2 kΩ resistor is removed from the circuit diagram of Fig. Q17. The resulting circuit diagram is shown in Fig. 42(a).

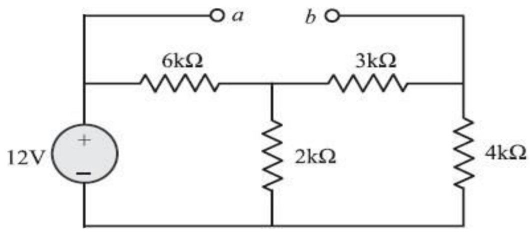


Fig. 42(a)

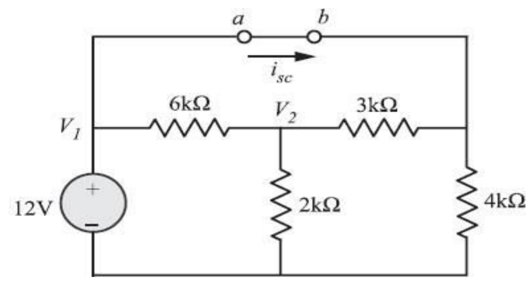


Fig. 42(b)

To find i_N or i_{sc}:

Refer Fig. 42(b). By inspection, $V_1 = 12\,V$

Applying KCL at node V_2:

$$\frac{V_2 - V_1}{6k} + \frac{V_2}{2k} + \frac{V_2 - V_1}{3k} = 0$$

Substituting V_1, we get $V_2 = 6\,V$

$$i_{sc} = \frac{V_1 - V_2}{3k} + \frac{V_1}{4k} = 5mA$$

To find R_N:

Deactivate all the independent sources. Refer Fig. 42(c).

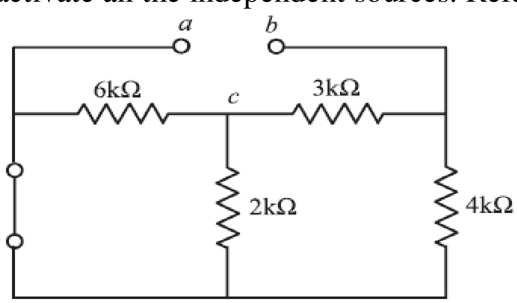


Fig. 42(c)

$$R_N = R_{ab} = 4k \,||\, (6k \,||\, 2k) + 3k] = 2.12k\Omega$$

Hence, the Norton equivalent circuit along with 2 kΩ resistor is as shown in Fig. 42(d)

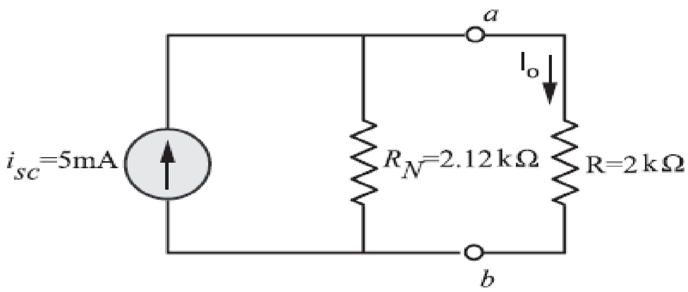


Fig. 42(d)

$$i_0 = \frac{i_{sc} \times R_N}{R_N + R} = 2.57mA$$

3. Refer the circuit shown in Fig.43. find the value of i_b using Norton's equivalent circuit. Take R = 667 Ω.

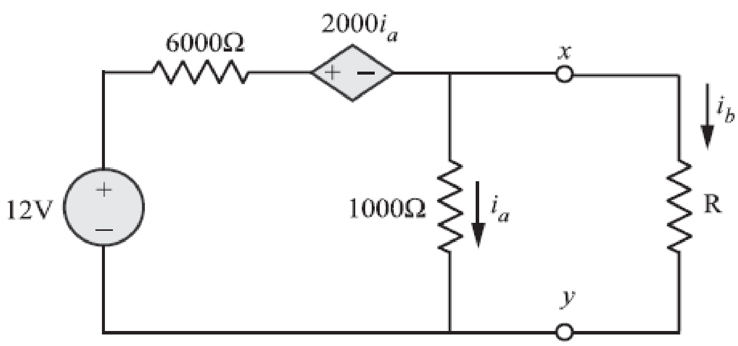


Fig.43

Solution:
Since we want the current flowing through R, remove R from the circuit of Fig.43. The resulting circuit diagram is shown in Fig. 43(a).

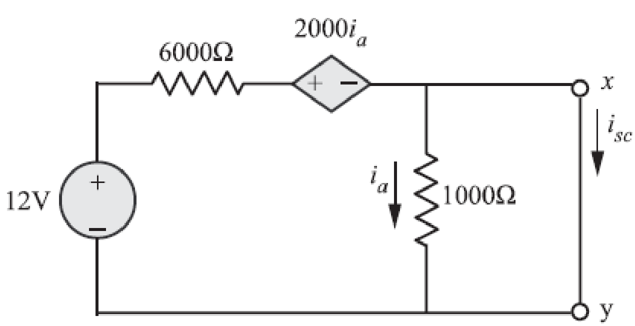


Fig.43(a)

Since i_a= 0A, $i_{sc} = \frac{12}{6000} = 2mA$

To find R_N:
The procedure for finding R_N is same that of R_t in the Thevenin equivalent circuit.
$R_t = R_N = \frac{v_{oc}}{i_{sc}}$

To find v_{oc}, make use of the circuit diagram shown in Fig.43(b). Do not deactivate any source.

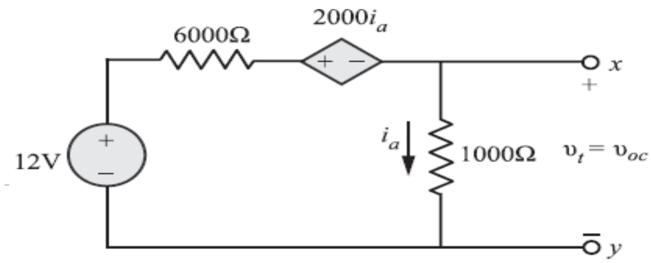


Fig.43(b)

Applying KVL clockwise, we get
$12 - 6000i_a - 2000i_a - 1000i_a = 0$
$i_a = 1.33\text{mA}$

$v_{oc} = i_a \times 1000 = 1.33\text{V}$
Therefore, $R_N = \frac{v_{oc}}{i_{sc}} = 667V$

The Norton equivalent circuit along with resistor R is as shown in Fig.43(c)

$i_b = \frac{i_{sc}}{2} = 1mA$

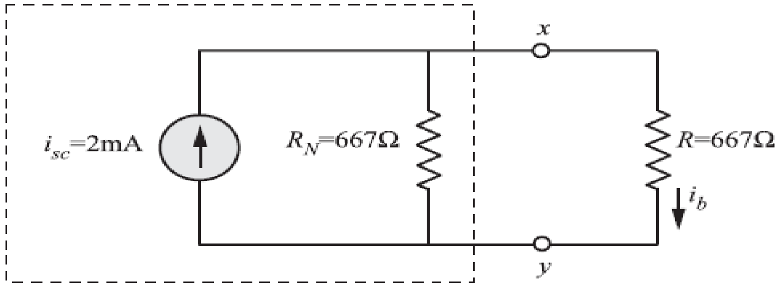


Fig. 43(c)

Self Assessment:

1. Find V_0 in the circuit of Fig.45

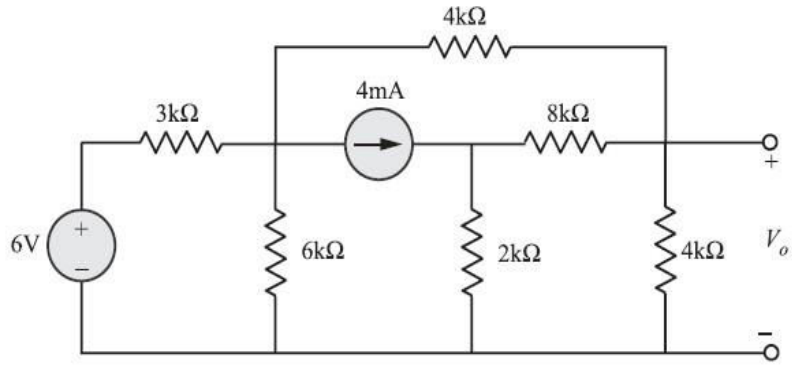


Fig. 45 (Ans : V_0 =258mV)

2. For the circuit shown in Fig.46, calculate the current in the 6Ω resistance using Norton's theorem.

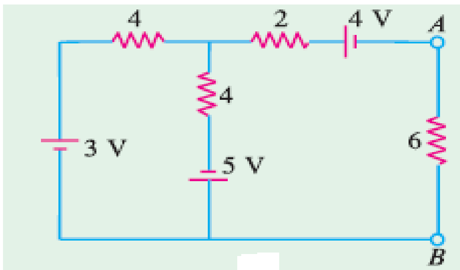


Fig.46

(Ans : 0.5A from B to A)

3. Find the Norton equivalent to the left of the terminals a-b for the circuit of Fig.47

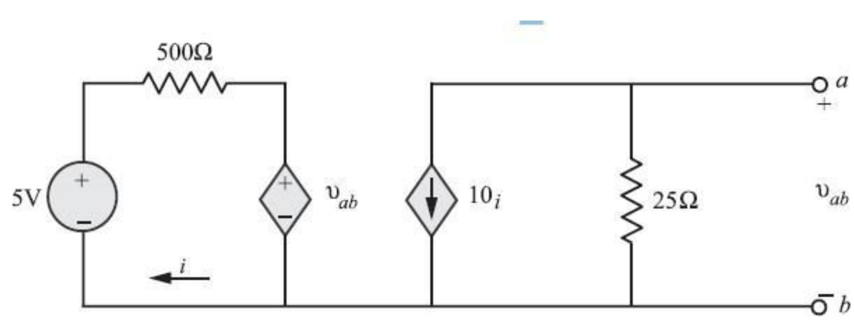


Fig.47 (Ans : i_{sc} =100mA, R_N=50Ω)

Maximum Power Transfer Theorem

In circuit analysis, we are sometimes interested in determining the maximum power that a circuit can supply to the load. Consider the linear circuit A as shown in Fig. 2.6.

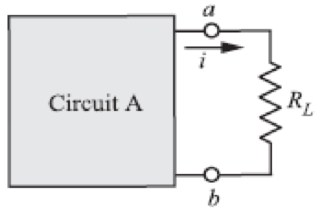


Fig. 2.6: A Linear circuit

Circuit A is replaced by its Thevenin's equivalent circuit as seen from a and b as shown in Fig. 2.7.

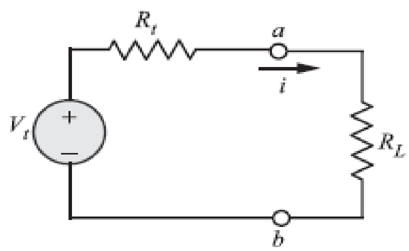


Fig. 2.7: Thevenin's equivalent circuit is substituted for circuit A

We wish to find the value of the load R_L such that the maximum power is delivered to it.

The power that is delivered to the load is given by

$p = i^2 R_L = \left(\frac{V_t}{R_t+R_L}\right)^2 \times R_L$ ....(i)

Assuming that V_t and R_L are fixed for a given source, the maximum power is a function of R_L. In order to determine the value of R_L that maximizes *p*, we differentiate *p* with respect to R_L and equate the derivative to zero.

$\frac{dp}{dR_L} = V_t^{\ 2}\left[\frac{(R_t+R_L)^2 - 2(R_t+R_L)}{(R_t+R_L)^2}\right] = 0$.................(ii)

which yields $R_L = R_t$

To confirm that equation (ii) is a maximum, it should be shown that $\frac{d^2p}{dR_L^{\ 2}} < 0$

Hence, maximum power is transferred to the load when R_L is equal to the Thevenin's equivalent resistance R_t.

The maximum power transferred to the load is obtained by substituting $R_L = R_t$ in equation (i).

Accordingly, $p_{max} = \frac{V_t^{\ 2} R_L}{(2R_L^{\ 2})} = \frac{V_t^{\ 2}}{4R_L}$

The maximum power transfer theorem states that the maximum power delivered by a source represented by its Thevenin equivalent circuit is attained when the load R_L is equal to the Thevenin's resistance R_t.

Problems

1. Find R_L for maximum power transfer and the maximum power that can be transferred in the network shown in Fig. 48.

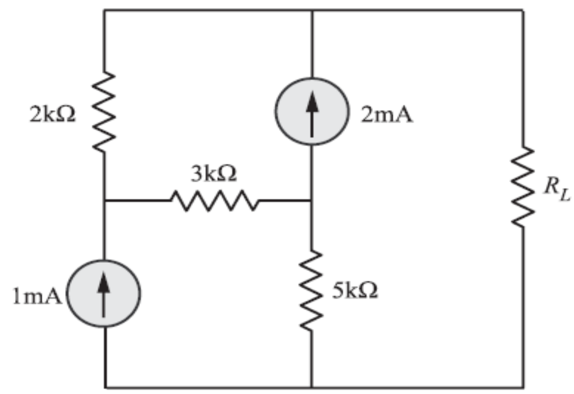


Fig.48

Solution:

Disconnect the load resistor R_L and deactivate all the independent sources to find R_t. The resultant circuit is as shown in the Fig.48(a)

Fig.48(a)

$R_t = R_{ab} = 10K\Omega$

For maximum power transfer,
$R_L = R_t = 10K\Omega$

Let us next find V_{oc} or V_t.

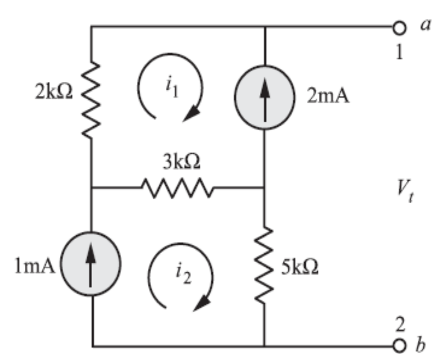


Fig.48(b)

Refer Fig.48(b)
By inspection,
$i_1 = -2mA$ & $i_2 = 1mA$

Applying KVL clockwise to the loop $5\ k\Omega \to 3k\Omega \to 2k\Omega \to a-b$, we get

$$-5k \times i_2 + 3k(i_1 - i_2) + 2k \times i_1 + V_t = 0$$

On solving, $V_t = 18\ V$

The Thevenin's equivalent circuit with load resistor R_L is as shown in Fig.48(c)

$$i = \frac{18}{20k} = 0.9mA$$

$$P_{max} = P_L = (0.9m)^2 \times 10K = 8.1mW$$

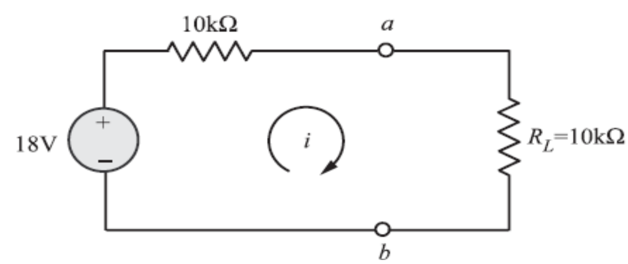


Fig.48(c)

2. Find the value of R_L for maximum power transfer for the circuit shown in Fig.49. Hence find P_{max}.

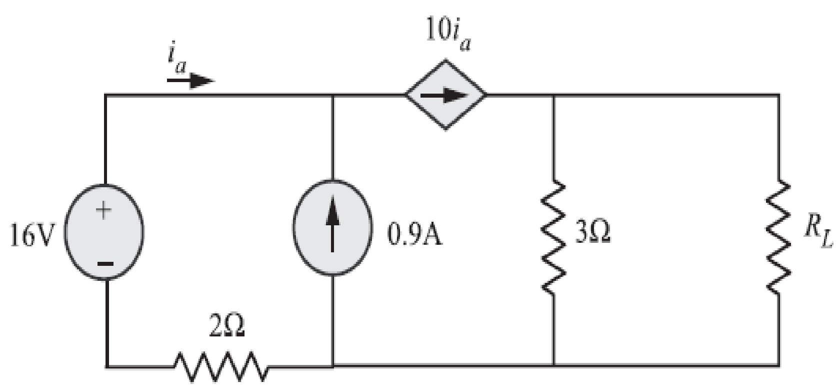


Fig. 49

Solution:

Removing R_L from the original circuit gives us the circuit diagram shown in Fig.49(a)

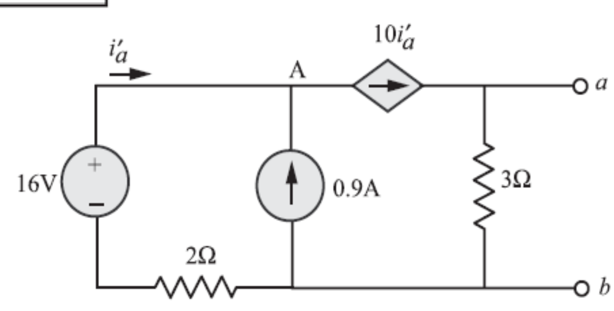


Fig.49(a)

$i'_a + 0.9 - 10i'_a = 0$

To find V_{oc} :

KCL at node A :

$i'_a = 0.1A$

Hence,
$V_{oc} = 3(10i'_a) = 3V$

To find R_t, refer Fig.49(b) we need to compute i_{sc} with all independent sources activated.

KCL at node A:

$i''_a + 0.9 - 10i''_a = 0$
$i''_a = 0.1A$

Hence, $i_{sc} = 10i''_a = 1A$

$R_t = \frac{V_{oc}}{i_{sc}} = 3\Omega$
Hence, for maximum power transfer
$R_L = R_t = 3\Omega$.
The Thevenin's equivalent circuit with $R_L = 3\Omega$ inserted between the terminals a-b gives the network shown in Fig.49(c).

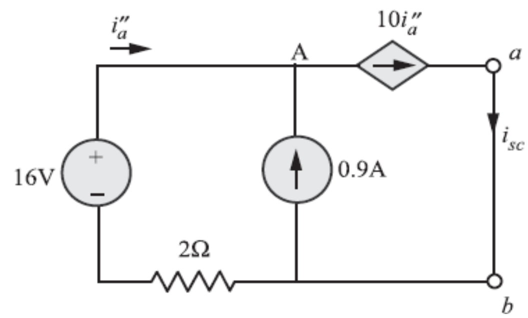


Fig.49(b)

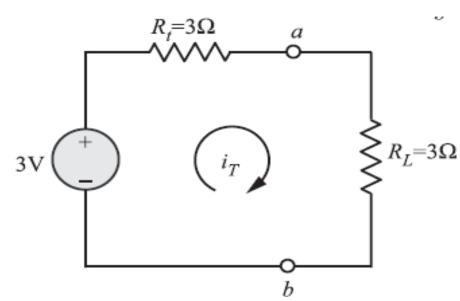


Fig.49(c).

$i_T = \frac{3}{6} = 0.5A$

$P_{max} = (0.5)^2 \times 3 = 0.75W$

Self Assessment

1. Find the value of R_L for maximum power transfer in the circuit shown in Fig.50. Also find P_{max}.

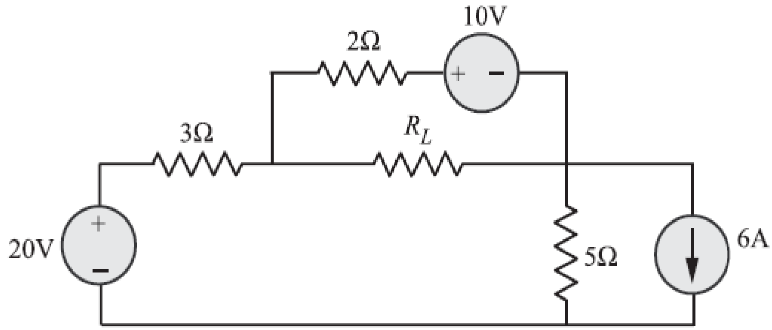


Fig.50

(Ans: P_{max} = 625mW)

2. Find the value of R_L in the network shown in Fig.51 that will achieve maximum power transfer, and determine the value of the maximum power.

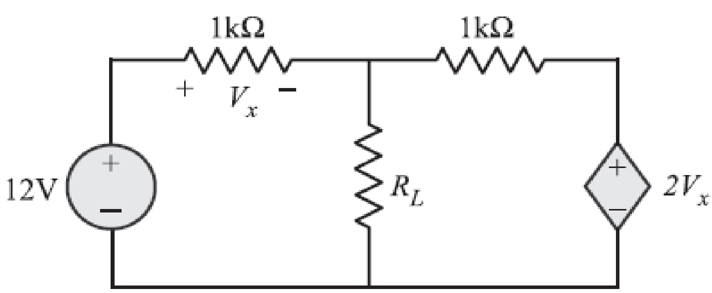


Fig.51

(Ans: P_{max} = 81mW)

3. Refer to the circuit shown in Fig.52
   (a) Find the value of R_L for maximum power transfer.
   (b) Find the maximum power that can be delivered to R_L.

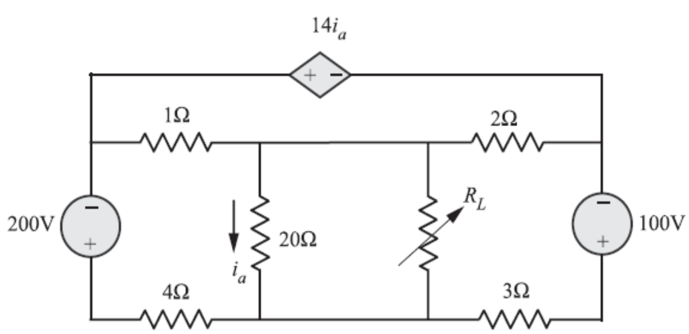


Fig.52

(Ans: R_L=2.5Ω, P_{max} = 2250W)

We have earlier shown that for a resistive network, maximum power is transferred from a source to the load, when the load resistance is set equal to the thevenin's resistance with thevenin's equivalent source. Now we extend this result to the ac circuits.

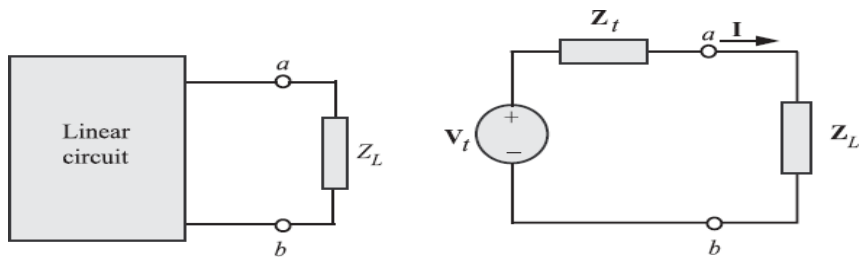


Fig. 2.8: (a)Linear circuit (b) Thevenin's equivalent circuit

In Fig. 2.8(a), the linear circuit is made up of impedances, independent and dependent sources.This linear circuit is replaced by its thevenin's equivalent circuit as shown in Fig. 2.8(b).
In rectangular form, the thevenin impedance Z_t and the load impedance Z_L are

$$Z_t = R_t + jX_t$$

and $$Z_L = R_L + jX_L$$

The current through the load is

$$I = \frac{V_t}{Z_t + Z_L} = \frac{V_t}{(R_t + jX_t) + (R_L + jX_L)}$$

The phasors I and V_t are the maximum values. The corresponding RMS values are obtained by dividing the maximum values by $\sqrt{2}$. Also, the RMS value of phasor current flowing in the load must be taken for computing the average power delivered to the load.
The average power delivered to the load is given by

$$P = \frac{1}{2}|I|^2 R_L$$

$$P = \frac{|V_t|^2 \frac{R_L}{2}}{(R_t + R_L)^2 + (X_t + X_L)^2} \qquad \text{(i)}$$

Our idea is to adjust the load parameters R_L and X_L so that P is maximum. To do this, we get $\frac{\partial P}{\partial R_L}$ and $\frac{\partial P}{\partial X_L}$ equal to zero.

$$\frac{\partial P}{\partial X_L} = \frac{-|V_t|^2 R_L(X_t + X_L)}{[(R_t + R_L)^2 + (X_t + X_L)^2]^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{|V_t|^2 (R_t + R_L)^2 + (X_t + X_L)^2 - 2R_L (R_t + R_L)}{2[(R_t + R_L)^2 + (X_t + X_L)^2]^2}$$

Setting $\frac{\partial P}{\partial X_L} = 0$ gives $X_L = -X_t$ (ii)

and setting $\frac{\partial P}{\partial R_L} = 0$ gives $R_L = \sqrt{R_t^2 + (X_t + X_L)^2}$ (iii)

Combining equations (ii) and (iii), we can conclude that for maximum average power transfer, Z_L must be selected such that $X_L = -X_t$ and $R_L = R_t$. That is the maximum average power of a circuit with an impedance Z_t that is obtained when Z_L is set equal to complex conjugate of Z_t .

Setting $X_L = -X_t$ and $R_L = R_t$ in equation (i), we get the maximum average power as

$$P = \frac{|V_t|^2}{8R_t}$$

In a situation where the load is purely real, the condition for maximum power transfer is obtained by putting $X_L = 0$ in equation (iii). That is,

$$R_L = \sqrt{R_t^2 + X_t^2} = |Z_t|$$

Hence for maximum average power transfer to a purely resistive load, the load resistance is equal to the magnitude of thevenin impedance.

Maximum average power can be delivered to Z_L only if $Z_L = Z_t^*$.
Z_t^* is the complex conjugate of Z_L

Problems

1. Find the load impedance that transfers the maximum power to the load for the circuit shown in Fig.53

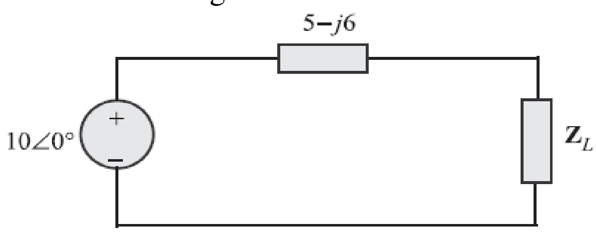
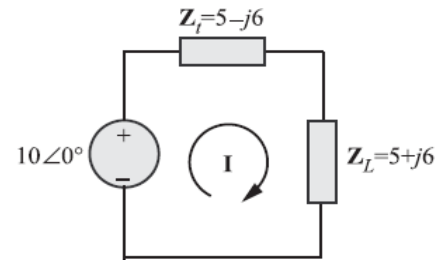


Fig.53

Solution:

We select, $Z_L = Z_t^*$ for maximum power transfer.

Hence $Z_L = 5 + j6$



2. For the circuit of Fig.54, what is the value of Z_L that will absorb the maximum average power?

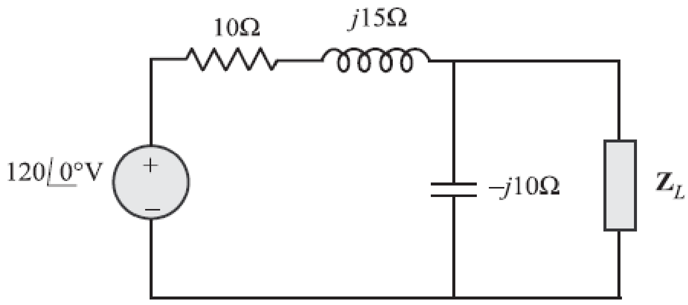


Fig.54

Solution:

Disconnecting Z_L from the original circuit we get the circuit as shown in Fig.54(a). The first step is to find V_t.

$$V_t = V_{oc} = i_1 \times -j10$$

$$V_t = \frac{120\angle 0°}{10 + j15 - j10} \times -j10$$

$$V_t = 107.33\angle -116.57°\ V$$

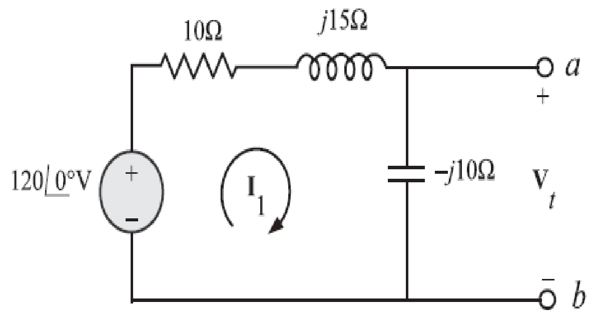


Fig.54(a).

The next step is to find Z_L. This requires deactivating the independent voltage source of Fig.54(b)

$$Z_t = (10 + j15) || -j10$$

$$Z_t = 8 - j14\Omega$$

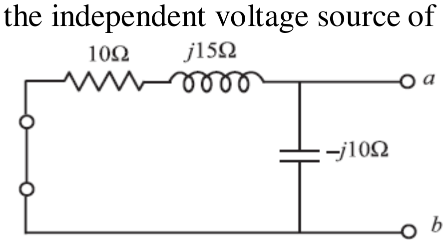


Fig.54(b)

The value of Z_L for maximum average power absorbed is

$Z_t^* = 8 + j14\Omega$

The Thevenin's equivalent circuit along with Z_L is as shown in Fig.54(c)

$Z_L = 8 + j14\Omega$

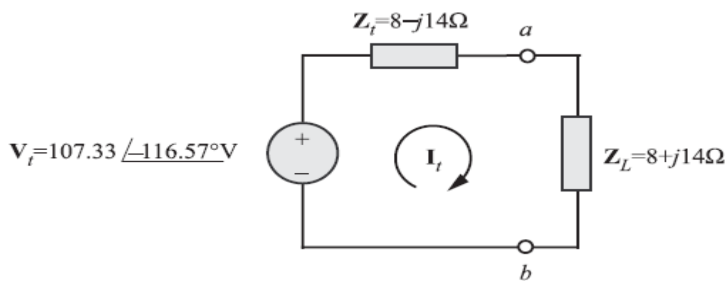


Fig.54(c)

Self Assessment

1. Find the load impedance that transfers the maximum average power to the load and determine the maximum average power transferred to the load Z_L shown in Fig.55.

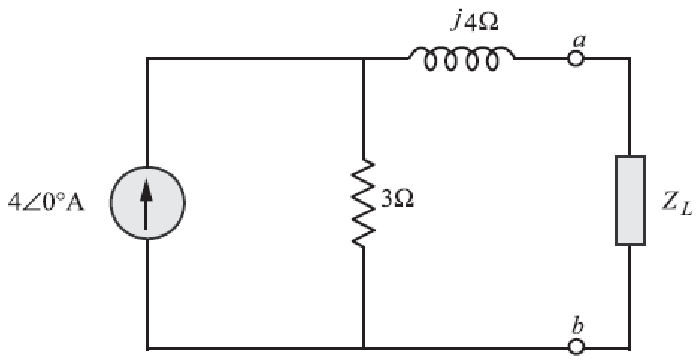


Fig.55

(Ans: P_{max}= 6W)

2.5 Reciprocity theorem

The reciprocity theorem states that in a linear bilateral single source circuit, the ratio of excitation to response is constant when the positions of excitation and response are interchanged.

Conditions to be met for the application of reciprocity theorem:
(i) The circuit must have a single source.
(ii) Dependent sources are excluded even if they are linear.
(iii) When the positions of source and response are interchanged, their directions should be marked same as in the original circuit.

Problems

1. In the circuit shown in Fig.57, find the current through 1.375 Ω resistor and hence verify reciprocity theorem.

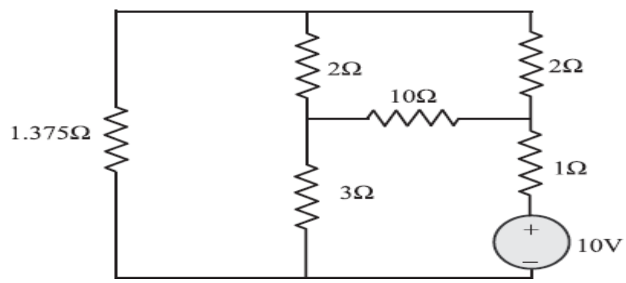


Fig.57

Solution:

Apply KVL to Fig.57(a)

KVL clockwise for mesh 1: KVL clockwise for mesh 2: KVL clockwise for mesh 3:

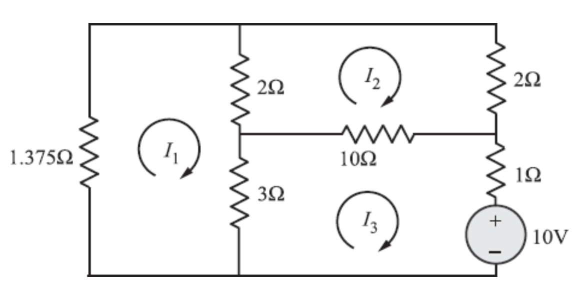


Fig.57(a)

$$6.375I_1 - 2I_2 - 3I_3 = 0$$
$$-2I_1 + 14I_2 - 10I_3 = 0$$
$$-3I_1 - 10I_2 + 14I_3 = -10$$

Using Cramer's rule, we get $I_1 = -2A$

Verification using reciprocity theorem:
The circuit is redrawn by interchanging the positions of excitation and response. The new circuit is shown in Fig.57(b)

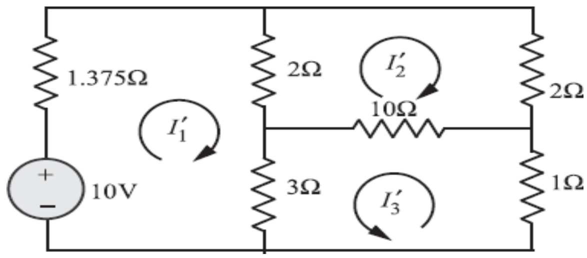


Fig.57(b)

1. TELLEGEN‟S THEOREM:

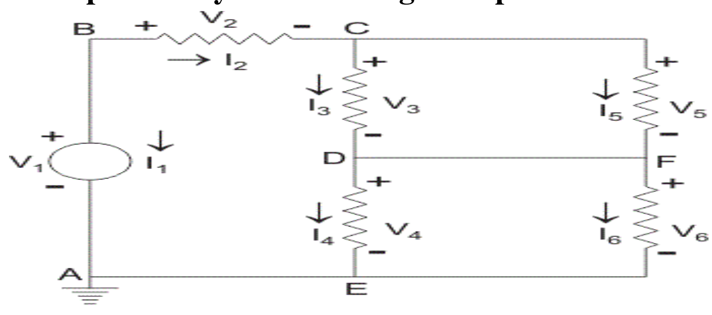
Dc Excitation:

Tellegen‘s theorem states algebraic sum of all delivered power must be equal to sum of all received powers.According to Tellegen‘s theorem, the summation of instantaneous powers for the n number of branches in an electrical network is zero. Are you confused? Let's explain. Suppose n number of branches in an electrical network have i1, i2, i3…. in respective instantaneous currents through them. These currents satisfy Kirchhoff's Current Law. Again, suppose these branches have instantaneous voltages across them are v1, v2, v3, vn respectively. Ifthese voltages across these elements satisfy Kirchhoff Voltage Law then,

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

v_k is the instantaneous voltage across the k^{th} branch and i_k is the instantaneous current flowing through this branch. Tellegen‟s theorem is applicable mainly in general class of lumped networks that consist of linear, non-linear, active, passive, time variant and time variant elements.

This theorem can easily be explained by the following example.



In the network shown, arbitrary reference directions have been selected for all of the branch currents, and the corresponding branch voltages have been indicated, with positive reference direction at the tail of the current arrow. For this network, we will assume a set of branch voltages satisfy the Kirchhoff voltage law and a set of branch current satisfy Kirchhoff current law at each node.We will then show that these arbitrary assumed voltages and currents satisfy the equation.

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

And it is the condition of Tellegen's theorem. In the network shown in the figure, let v_1, v_2 and v_3 be 7, 2 and 3 volts respectively. Applying Kirchhoff Voltage Law around loop ABCDEA. We see that v_4 = 2 volt is required. Around loop CDFC, v_5 is required to be 3 volt and around loop DFED, v_6 is required to be 2. We next apply Kirchhoff's Current Law successively to nodes B, C and D. At node B let i_i = 5 A, then it is required that i_2 = - 5 A. At node C let i_3 = 3 A and then i_5 is required to be - 8. At node D assume i_4 to be 4 then i_6 is required to be - 9. Carrying out the operation of equation,

We get,

$$7 \times 5 + 2 \times (-5) + 3 \times 3 + 2 \times 4 + 3 \times (-8) + 2 \times (-9) = 0$$

Hence Tellegen's theorem is verified.

Conclusion:

With help of Network Theorems one can find the choice of load resistance RL that results in the maximum power transfer to the load. On the other hand, the effort necessary to solve this problem-using node or mesh analysis methods can be quite complex and tedious from computational point of view.

Reference:

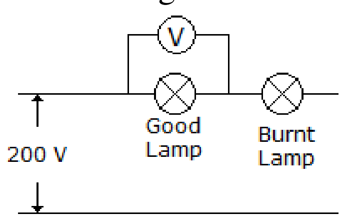
[1].Sudhakar, A., Shyammohan, S. P.; “Circuits and Network”; Tata McGraw-Hill New Delhi,2000
[2]. A William Hayt, “Engineering Circuit Analysis” 8th Edition, McGraw-Hill Education 2004
[3]. Paranjothi SR, “Electric Circuits Analysis,” New Age International Ltd., New Delhi, 1996.

Post Test MCQs:

1.The equation $\sum_{k=1}^{b} v_k i_k = 0$ where v_1, v_2 ... v_b are the instantaneous branch voltages and i_1, i_2 ... i_b are the instantaneous branch currents pertains to

a. Compensation theorem
b. Superposition theorem
c. Tellegen's theorem
d. Reciprocity theorem

2.The reading of the voltmeter in figure will be __________ volt.

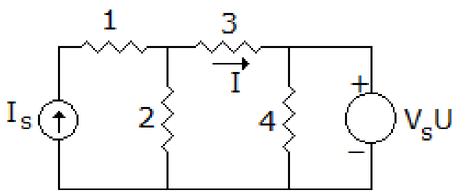


a. 100
b. 150
c. 0
d. 175

3.A circuit is replaced by its Thevenin's equivalent to find current through a certain branch. If V_{TH} = 10 V and R_{TH} = 20 Ω, the current through the branch

a. will always be 0.5 A
b. will always be less than 0.5 A
c. will always be equal to or less than 0.5 A
d. may be 0.5 A or more or less

4. Given I_s = 20 A, V_s = 20 V, the current I in the 3 Ω resistance is given by



a. 4 A
b. 8 A
c. 8 A
d, 16 A

5. For maximum transfer of power, internal resistance of the source should be

a. equal to load resistance
b. less than the load resistance
c. greater than the load resistance
d. none of the above

6.Tellegen's theorem is applicable to

a. circuits with passive elements only
b. circuits with linear elements only
c. circuits with time invariant elements only
d. .circuits with active or passive, linear or nonlinear and time invariant or time varying elements

7. **Assertion (A):** Millman's theorem helps in replacing a number of current sources in parallel by a single current source.
Reason (R): Maximum power transfer theorem is applicable only for dc, circuits.

a. Both A and R are true and R is correct explanation of A
b. Both A and R are true and R is not the correct explanation of A
c. .A is true but R is false
d. A is false but R is true

8. “Any number of current sources in parallel may be replaced by a single current source whose current is the algebraic sum of individual currents and source resistance is the parallel

combination of individual source resistances".
The above statement is associated with.

a. Thevenin's theorem
b. .Millman's theorem.
c. Maximum power transfer theorem
d. None of the above.

9. Superposition theorem can be applied only to circuits having

a. Resistive elements
b. Passive elements
c. Nonlinear elements
d. .Linear bilateral elements

10. A nonlinear network does not satisfy

a. Superposition condition
b. Homogeneity condition
c. Both homogeneity as well as superposition condition
d. Homogeneity, superposition and associative condition.

UNIT III
TWO PORT NETWORKS AND FILTERS DESIGN

AIM:

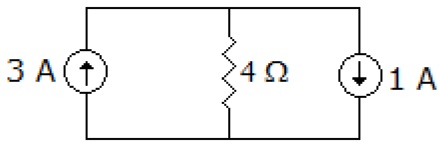
To analyze the behavior of the circuit's response in time domain

Pre-Requisites:

Knowledge of Basic Mathematics – II & Basic Electronics Engineering

Pre - MCQs:

1.In figure, the power associated with 3 A source is



a. 36 W

b. 24 W

c. 16 W

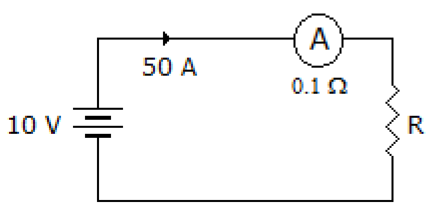
d. 8 W

Answer: Option **B**

Explanation:

Voltage across 4Ω resistance = source voltage = 8V. Power = 8 X 3 = 24 W.

2. In figure, the value of R should be



a. 0.2 Ω

b. 0.1 Ω

c. 0.05 Ω

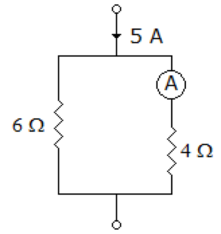
d. 0.1 Ω

Answer: Option **B**

Explanation:

$$\frac{10}{0.1 + R} = 50 \text{ or } R = 0.1\ \Omega$$

3. In figure, A is ideal ammeter having zero resistance. It will read __________ ampere.



a. 2

b2.5

c. 3

d.4

Answer: Option **C**

Explanation:

$$I = \frac{5 \times 6}{6 + 4} \Rightarrow 3\ A.$$

INTRODUCTION:

A pair of terminals through which a current may enter or leave a network is known as a *port*. Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks. Most of the circuits we have dealt with so far are two-terminal or one-port circuits, represented in Figure 2(a). We have considered the voltage across or current through a single pair of terminals—such as the two terminals of a resistor, a capacitor, or an inductor. We have also studied four-terminal or two-port circuits involving op amps, transistors, and transformers, as shown in Figure 2(b). In general, a network may have *n* ports. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.

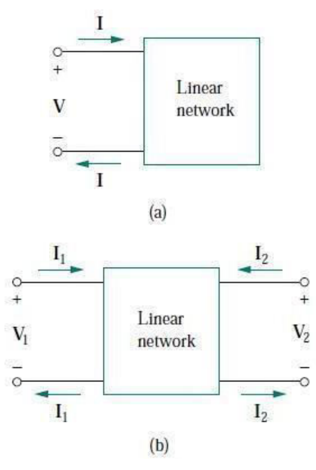
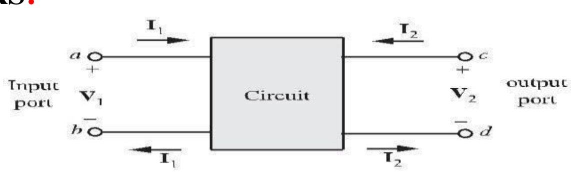


Figure 2: (a) One-port network, (b) two-port network.

A pair of terminals through which a current may enter or leave a network is known as a port. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero. There are several reasons why we should study two-ports and the parameters that describe them. For example, most circuits have two ports. We may apply an input signal in one port and obtain an output signal from the other port. The parameters of a two-port network completely describe its behavior in terms of the voltage and current at each port. Thus, knowing the parameters of a two port network permits us to describe its operation when it is connected into a larger network. Two-port networks are also important in modeling electronic devices and system components. For example, in electronics, two-port networks are employed to model transistors and Op-amps. Other examples of electrical components modeled by two-ports are transformers and transmission lines.

Four popular types of two-port parameters are examined here: impedance, admittance, hybrid, and transmission. We show the usefulness of each set of parameters, demonstrate how they are related to each other

IMPEDANCE PARAMETERS:



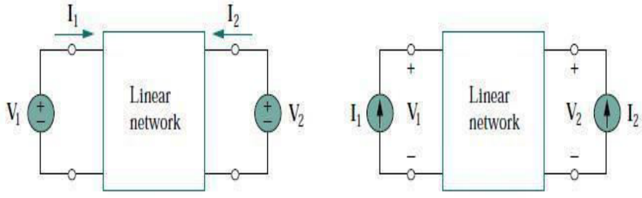
Impedance and admittance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks. We discuss impedance parameters in this section and admittance parameters in the next section.

A two-port network may be voltage-driven as in Figure 3 (a) or current-driven as in Figure 3(b). From either Figure 3(a) or (b), the terminal voltages can be related to the terminal currents as

$$V1=Z_{11}I_1+Z_{12}I_2$$

$$V2=Z_{21}I_1+Z_{22}I_2$$

Where the z terms are called the impedance parameters, or simply z parameters, and have units of ohms.



(a) (b)

The values of the parameters can be evaluated by setting $\mathbf{I}_1 = 0$ (input port open-circuited) or $\mathbf{I}_2 = 0$ (output port open-circuited).

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

Since the z parameters are obtained by open-circuiting the input or output port, they are also called the open-circuit impedance parameters. Specifically,

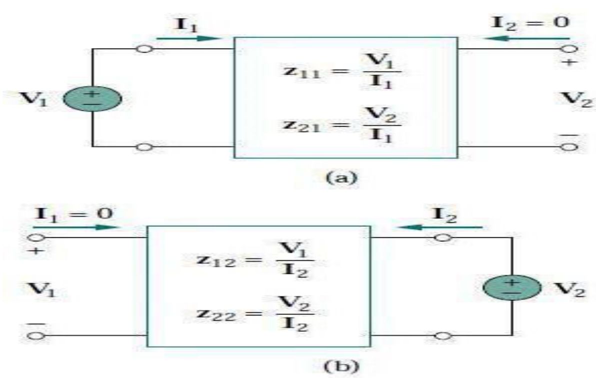
$\mathbf{z}_{11}$ = Open-circuit input impedance

$\mathbf{z}_{12}$ = Open-circuit transfer impedance from port 1 to port 2 $\mathbf{z}_{21}$

= Open-circuit transfer impedance from port 2 to port 1 $\mathbf{z}_{22}$ =

Open-circuit output impedance

We obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ by connecting a voltage $\mathbf{V}_1$ (or a current source $\mathbf{I}_1$) to port 1 with port 2 open-circuited as in Figure 4 and finding $\mathbf{I}_1$ and $\mathbf{V}_2$; we then get



Determination of the *z* parameters: (a) finding $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ (b) finding $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$.

$Z_{11}=V_1/I_1$, $Z_{21}=V_2/I_1$

We obtain $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$ by connecting a voltage $\mathbf{V}_2$ (or a current source $\mathbf{I}_2$) to port 2 with port 1 open-circuited as in Figure 4) and finding $\mathbf{I}_2$ and $\mathbf{V}_1$; we then get

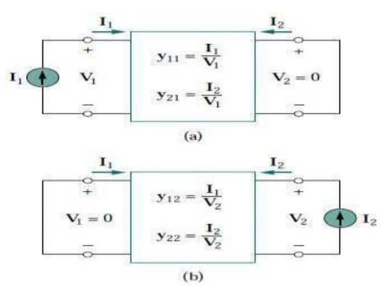
$Z_{12}=V_1/I_2$, $Z_{21}=V_2/I_2$

The above procedure provides us with a means of calculating or measuring the *z* parameters. Sometimes $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ are called driving-point impedances, while $\mathbf{z}_{21}$ and $\mathbf{z}_{12}$ are called *transfer impedances*. A driving-point impedance is the input impedance of a two-terminal (one-port) device. Thus, $\mathbf{z}_{11}$ is the input driving-point impedance with the output port open-circuited, while $\mathbf{z}_{22}$ is the output driving-point impedance with the input port open circuited.

When $z_{11} = z_{22}$, the two-port network is said to be symmetrical. This implies that the network has mirror like symmetry about some center line; that is, a line can be found that divides the network into two similar halves. When the two-port network is linear and has no dependent sources, the transfer impedances are equal ($z_{12} = z_{21}$), and the two-port is said to be reciprocal. This means that if the points of excitation and response are interchanged, the transfer impedances remain the same. A two-port is reciprocal if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.

ADMITTANCE PARAMETERS:

In the previous section we saw that impedance parameters may not exist for a two-port network. So there is a need for an alternative means of describing such a network. This need is met by the second set of parameters, which we obtain by expressing the terminal currents in terms of the terminal voltages. In either Figure 5(a) or (b), the terminal currents can be expressed in terms of the terminal voltages as



Determination of the y parameters: (a) finding y_{11} and y_{21}, (b) finding y_{12} and y_{22}.

$$I_1=Y_{11}V_1+Y_{12}V_2$$

$$I_2=Y_{21}V_1+Y_{22}V_2$$

The **Y** terms are known as the admittance parameters (or, simply, y parameters) and have units of Siemens

The values of the parameters can be determined by setting **V**1 = 0 (input port short-circuited) or **V**2 = 0 (output port short-circuited). Thus,

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$
$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

Since the *y* parameters are obtained by short-circuiting the input or output port, they are also called the short-circuit admittance parameters. Specifically,

$\mathbf{y}_{11}$ = Short-circuit input admittance

$\mathbf{y}_{12}$ = Short-circuit transfer admittance from port 2 to port 1

$\mathbf{y}_{21}$ = Short-circuit transfer admittance from port 1 to port 2

$\mathbf{y}_{22}$ = Short-circuit output admittance

We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by connecting a current $\mathbf{I}_1$ to port 1 and short-circuiting port 2 and finding V1And $\mathbf{I}_2$.

Similarly, we obtain $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$ by connecting a current source $\mathbf{I}_2$ to port 2 and short-circuiting port 1 and finding $\mathbf{I}_1$ and $\mathbf{V}_2$, and then getting

This procedure provides us with a means of calculating or measuring the *y* parameters. The impedance and admittance parameters are collectively referred to as admittance parameters

HYBRID PARAMETERS:

The *z* and *y* parameters of a two-port network do not always exist. So there is a need for developing another set of parameters. This third set of parameters is based on making **V**1 and **I**2 the dependent variables. Thus, we obtain

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

Or in matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The **h** terms are known as the hybrid parameters (or, simply, h parameters) because they are a hybrid Combination of ratios. They are very useful for describing electronic devices such as transistors; it is much easier to measure experimentally the *h* parameters of such devices than to measure their *z* or *y* parameters. The hybrid parameters are as follows.

It is evident that the parameters $\mathbf{h}_{11}$, $\mathbf{h}_{12}$, $\mathbf{h}_{21}$, and $\mathbf{h}_{22}$ represent impedance, a voltage gain, a current gain, and admittance, respectively. This is why they are called the hybrid parameters. To be specific,

$\mathbf{h}_{11}$ = Short-circuit input impedance

$\mathbf{h}_{12}$ = Open-circuit reverse voltage gain

$\mathbf{h}_{21}$ = Short-circuit forward current gain

$\mathbf{h}_{22}$ = Open-circuit output admittance

The procedure for calculating the *h* parameters is similar to that used for the *z* or *y* parameters. We apply a voltage or current source to the appropriate port, short-circuit or open-circuit the other port, depending on the parameter of interest, and perform regular circuit analysis.

TRANSMISSION PARAMETERS:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

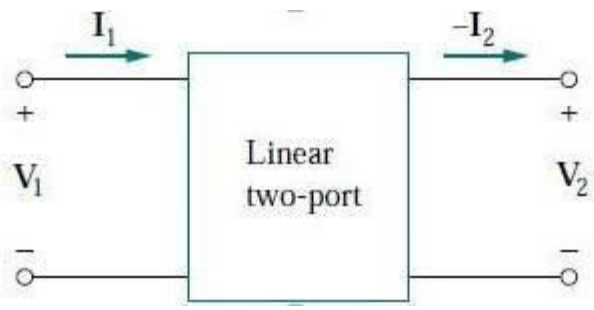
Since there are no restrictions on which terminal voltages and currents should be considered independent and which should be dependent variables, we expect to be able to generate many sets of parameters. Another set of parameters relates the variables at the input port to those at the output port.

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Thus,

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$
$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

The above Equations are relating the input variables ($\mathbf{V}_1$ and $\mathbf{I}_1$) to the output variables ($\mathbf{V}_2$ and $-\mathbf{I}_2$). Notice that in computing the transmission parameters, $-\mathbf{I}_2$ is used rather than $\mathbf{I}_2$, because the current is considered to be leaving the network, as shown in Figure 6. This is done merely for conventional reasons; when you cascade two-ports (output to input), it is most logical to think of $\mathbf{I}_2$ as leaving the two-port. It is also customary in the power − industry to consider $\mathbf{I}_2$ as leaving the two-port.



Terminal variables used to define the **ABCD** parameters.

The two-port parameters in above equations provide a measure of how a circuit transmits voltage and current from a source to a load. They are useful in the analysis of transmission lines (such as cable and fiber) because they express sending-end variables ($\mathbf{V}_1$ and $\mathbf{I}_1$) in terms of the receiving-end variables ($\mathbf{V}_2$ and $-\mathbf{I}_2$). For this reason, they are called transmission parameters. They are also known as **ABCD** parameters. They are used in the design of telephone systems, microwave networks, and radars.

The transmission parameters are determined as

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{B} = -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{D} = -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

Thus, the transmission parameters are called, specifically,

A = Open-circuit voltage ratio

B = Negative short-circuit transfer impedance

C = Open-circuit transfer admittance

D = Negative short-circuit current ratio

A and **D** are dimensionless, **B** is in ohms, and **C** is in Siemens. Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.

Condition of symmetry:

A two port network is said to be symmetrical if the ports can be interchanged without port voltages and currents

Condition of reciprocity:

A two port network is said to be reciprocal, if the rate of excitation to response is invariant to an interchange of the position of the excitation and response in the network. Network containing resistors, capacitors and inductors are generally reciprocal

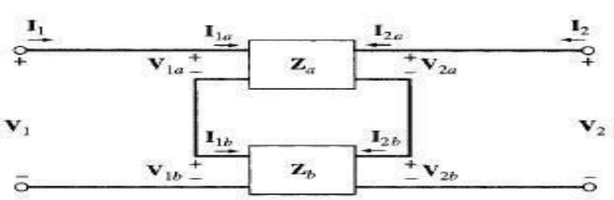
Condition for reciprocity and symmetry in two port parameters:

In Z parameters a network is termed to be reciprocal if the ratio of the response to the excitation remains unchanged even if the positions of the response as well as the excitation are interchanged.
A two port network is said to be symmetrical it the input and the output port can be interchanged without altering the port voltages or currents.

Parameter	Condition for reciprocity	Condition for symmetry
Z	Y Z_{12} Z_{21}	Y Z_{22} Z_{11}
Y	Y 12 21	Y 11 22
h	h_{11} h_{21}	h_{11} h_{21}
ABCD	AD - BC = 1	A = D

Interconnecting Two-Port Networks:

Two-port networks may be interconnected in various configurations, such as series, parallel, or cascade connection, resulting in new two-port networks. For each configuration, certain set of parameters may be more useful than others to describe the network. A series connection of two two-port networks a and b with open-circuit impedance parameters Za and Zb, respectively. In this configuration, we use the Z-parameters since they are combined as a series connection of two impedances.



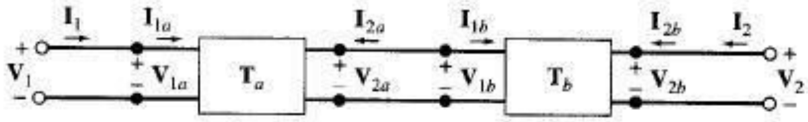
The Z-parameters of the series connection are Z 11= Z11A + Z11B

Or in the matrix form [Z]=[ZA]+[ZB]

Parallel Connection

[Y] = [YA] + [YB]

Cascade Connection



RELATIONSHIPS BETWEEN PARAMETERS:

Since the six sets of parameters relate the same input and output terminal variables of the same two-port network, they should be interrelated. If two sets of parameters exist, we can relate one set to the other set. Let us demonstrate the process with two examples.

Given the *z* parameters, let us obtain the *y* parameters.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

	z		y		h		g	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$-\frac{g_{12}}{\Delta_g}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	g_{11}	g_{12}
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}

FILTERS

PASSIVE FILTERS:

Frequency-selective or filter circuits pass to the output only those input signals that are in a desired range of frequencies (called pass band). The amplitude of signals outside this range of frequencies (called stop band) is reduced (ideally reduced to zero). Typically in these circuits, the input and output currents are kept to a small value and as such, the current transfer function is not an important parameter. The main parameter is the voltage transfer function in the frequency domain, Hv (jω) = Vo/Vi. As Hv (jω) is complex number, it has both a magnitude and a phase, filters in general introduce a phase difference between input and output signals. To minimize the number of subscripts, hereafter, we will drop subscript v of Hv. Furthermore, we concentrate on the ‖open-loop‖ transfer functions, Hvo, and denote this simply by H(jω).

Low-Pass Filters:

An ideal low-pass filter‘s transfer function is shown. The frequency between the pass- and-stop bands is called the cut-off frequency (ωc). All of the signals with frequencies below ωc are transmitted and all other signals are stopped.

In practical filters, pass and stop bands are not clearly defined, |H(jω)| varies continuously from its maximum toward zero. The cut-off frequency is, therefore, defined as the frequency at which |H(jω)| is reduced to 1/√2 =0.7 of its maximum value. This corresponds to signal power being reduced by 1/2 as P ∝ V 2.

Band-pass filters:

A band pass filter allows signals with a range of frequencies (pass band) to pass through and attenuates signals with frequencies outside this range.

Constant – K Low Pass Filter:

A network, either *T* or \[\pi\], is said to be of the constant-*k* type if Z_1 and Z_2 of the network satisfy the relation

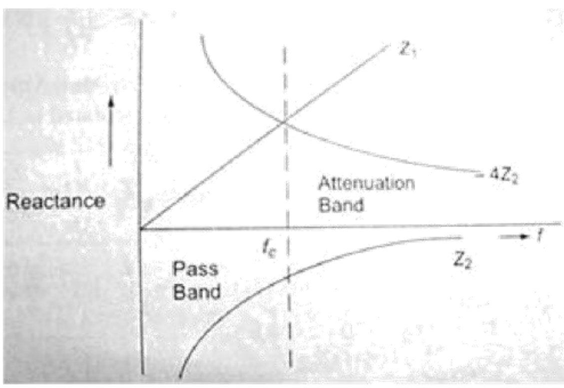
$$Z_1 Z_2 = k^2$$

Where Z_1 and Z_2 are impedance in the T and [pi] sections as shown in Fig. Equation 17.20 states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. *k* is a real constant, that is the resistance. *k* is often termed as design impedance or nominal impedance of the constant *k*-filter.

The constant *k, T* or π type filter is also known as the prototype because other more complex networks can be derived from it. Where $Z_1 = j\omega_L$ and $Z_2 = 1/j\omega_C$. Hence $Z_1Z_2 = {L \over C}={k^2}$ which is independent of frequency

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig.30.2. The cut-off frequency at the intersection of the curves Z_1 and $4Z_2$ is indicated as f_c. On the X-axis as

$Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1=-4Z_2$.



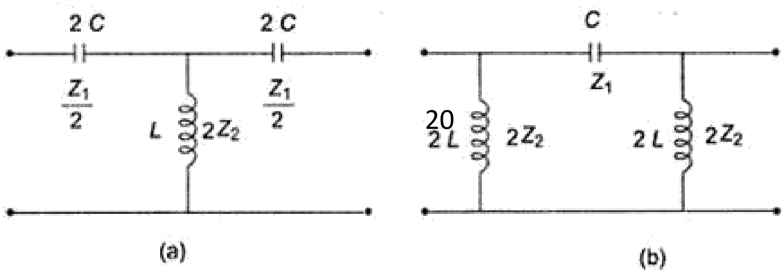
All the frequencies above f_c lie in a stop or attenuation band

The characteristic impedance of a π-network is given by

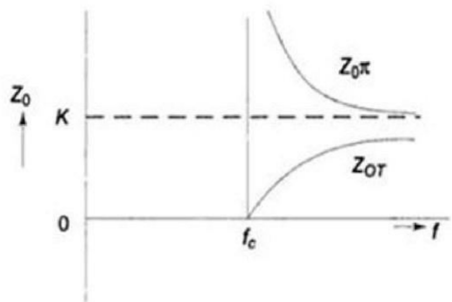
$$Z_{0\pi} = \frac{Z_1Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \qquad (30.5)$$

Constant K-High Pass Filter:

Constant K-high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig.30.1. The prototype high pass filters are shown in Fig.30.5, where $Z_1 = -j/\omega_C$ and $Z_2 = j\omega L$.

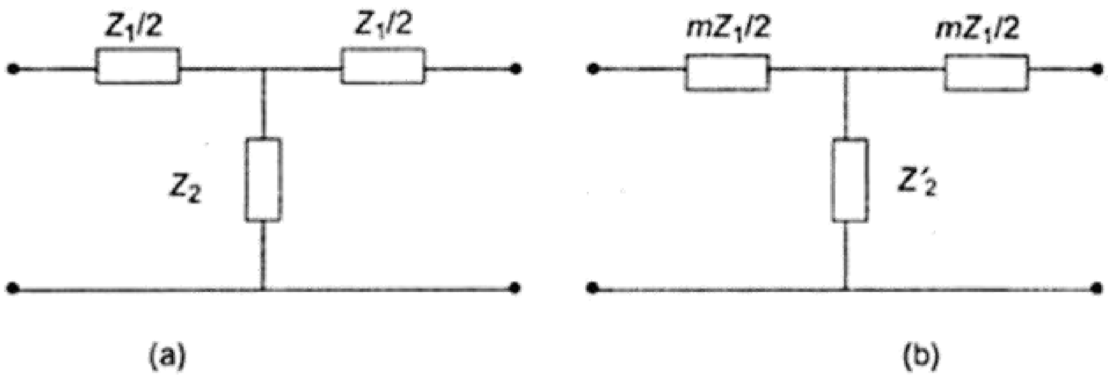


Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type. The plot of characteristic impedance with respect to frequency is shown



m-Derived T-Section:

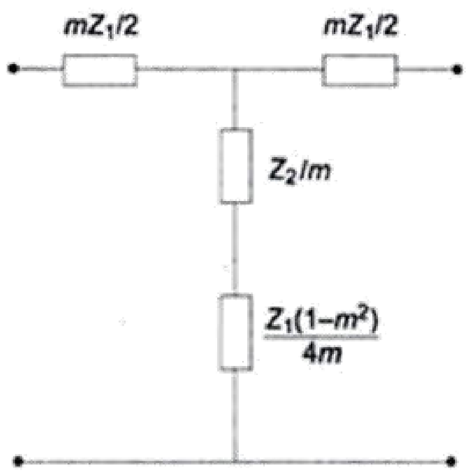
It is clear from previous chapter Figs 30.3 & 30.7 that the attenuation is not sharp in the stop band for k-type filters. The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedance be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band. However, cascading is not a proper solution from a practical point of view. This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of _a‘in the pass band. If the constant k section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called m-derived filter. Suppose a prototype T-network shown in Fig.31.1 (a) has the series arm modified as shown in Fig.31.1 (b), where m is a constant. Equating the characteristic impedance of the networks in us has



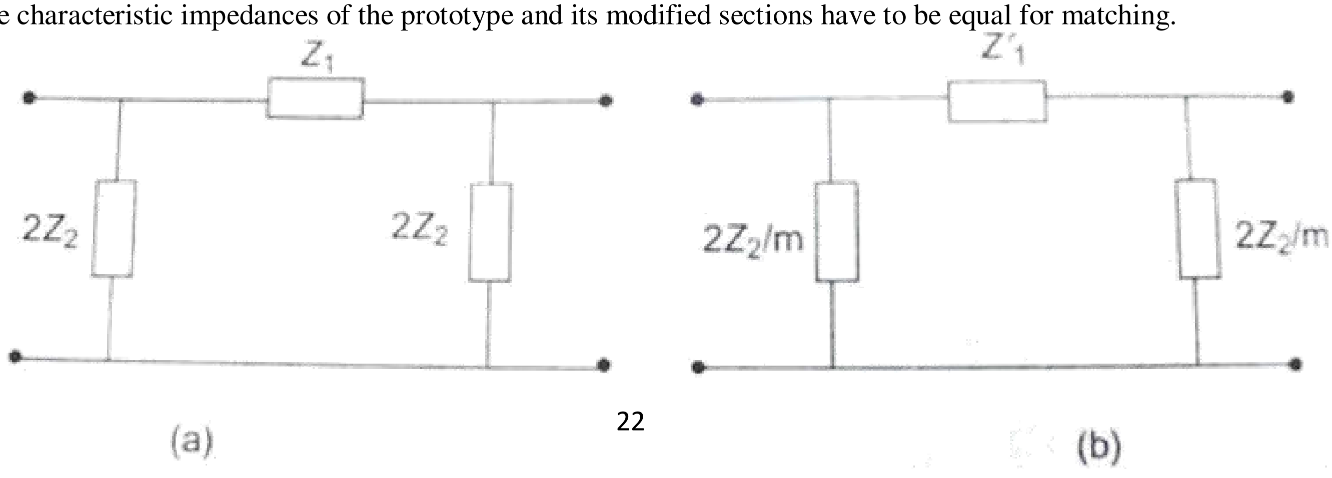
$Z_{0T} = Z_{0T}$

Where $Z_{0T'}$ is the characteristic impedance of the modified (m-derived) T-network.

Thus m-derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to \[\pi\] section network. Suppose a prototype p-network shown in Fig.31.3 (a) has the shunt arm modified as shown in Fig.31.3 (b).



The characteristic impedances of the prototype and its modified sections have to be equal for matching.



The characteristic impedance of the modified (m-derived) \[\pi\]-network

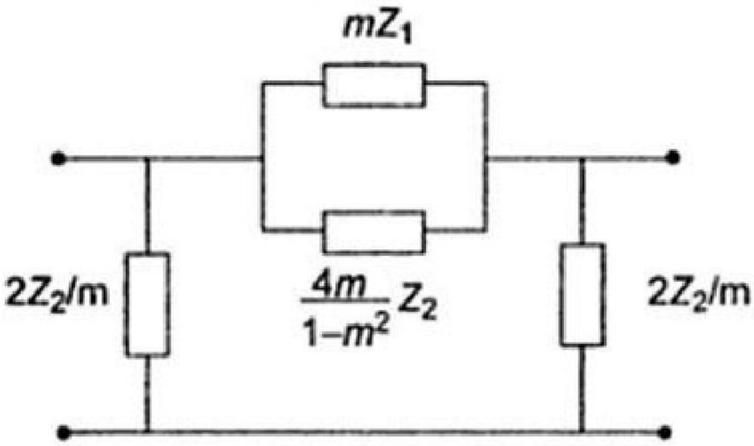
$$\therefore \quad \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1' \frac{Z_2}{m}}{1+\frac{Z_1'}{4.Z_2/m}}}$$

Or

$$Z_1' = \frac{Z_1 Z_2}{\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{mZ_1}{4}}$$

$$= \frac{Z_1 Z_2}{\frac{Z_2}{m} + \frac{Z_1}{4m}\left(1-m^2\right)}$$

$$Z_1' = \frac{Z_1 Z_2 \frac{4m^2}{\left(1-m^2\right)}}{\frac{Z_2 4m_2}{m\left(1-m_2\right)} + Z_1 m} = \frac{mZ_1 \frac{Z_2 4m}{\left(1-m^2\right)}}{mZ_1 + \frac{Z_2 4m}{\left(1-m^2\right)}} \quad \text{...}(31.2)$$

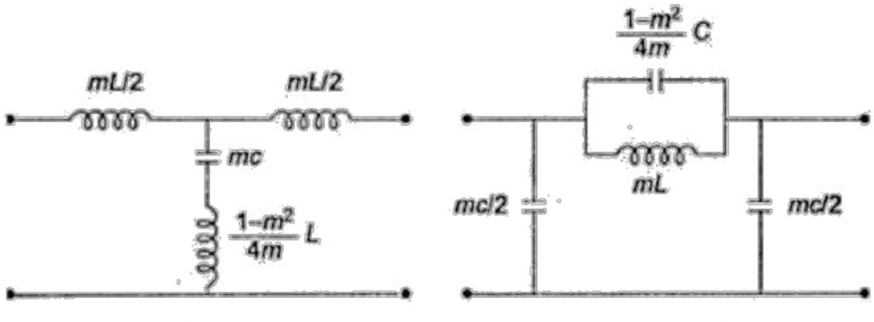


The series arm of the m-derived \[\pi\] section is a parallel combination of mZ_1 and $4mZ_2/1\text{-}m^2$

m-Derived Low Pass Filter:

In Fig.31.5, both m-derived low pass T and \[\pi\] filter sections are shown. For the T-section shown Fig.31.5

(a) The shunt arm is to be chosen so that it is resonant at some frequency f_x above cut-off frequency f_c its impedance will be minimum or zero. Therefore, the output is zero and will correspond to infinite attenuation at this particular frequency.



(a) (b)

$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4M}\right)\omega_r C}$$

$$\omega_r^2 = \frac{4}{LC\left(1-m^2\right)}$$

$$f_r = \frac{1}{\pi\sqrt{LC\left(1-m^2\right)}}$$

$$\therefore \qquad \alpha = 2\cosh^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_\alpha}\right)^2}}$$

And $$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_c}\right)^2(1=m)^2}}$$

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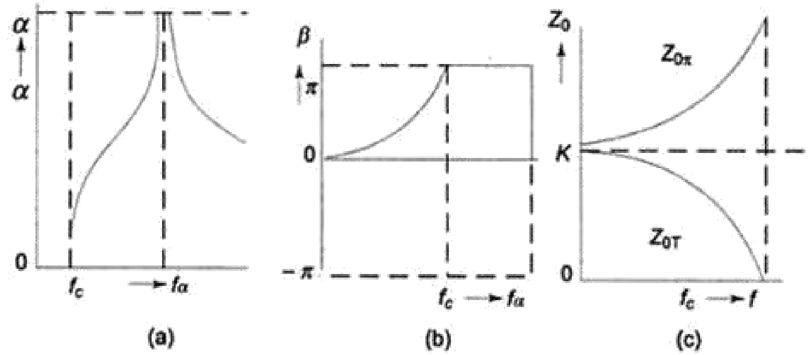
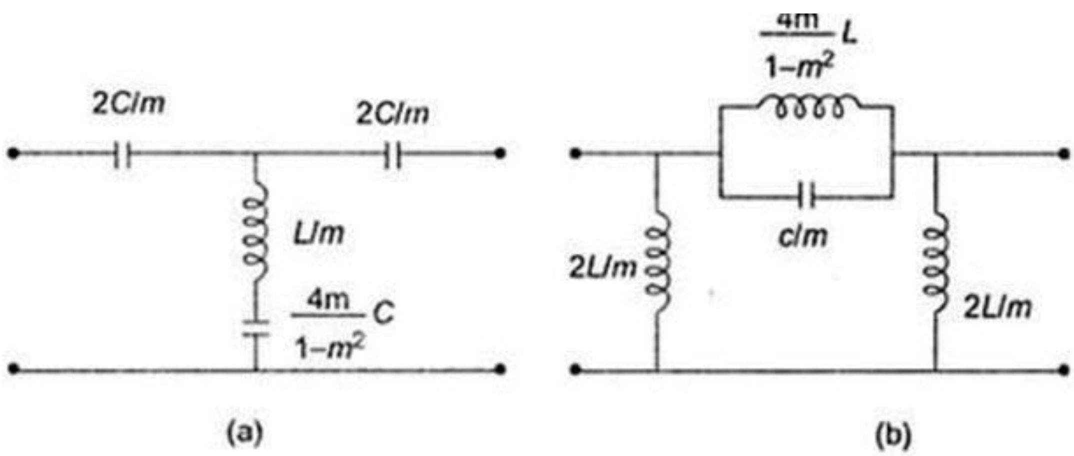
The variation of attenuation for a low pass m-derived section can be verified



m-derived High Pass Filter:

If the shunt arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and, thus, at resonance frequency or the frequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

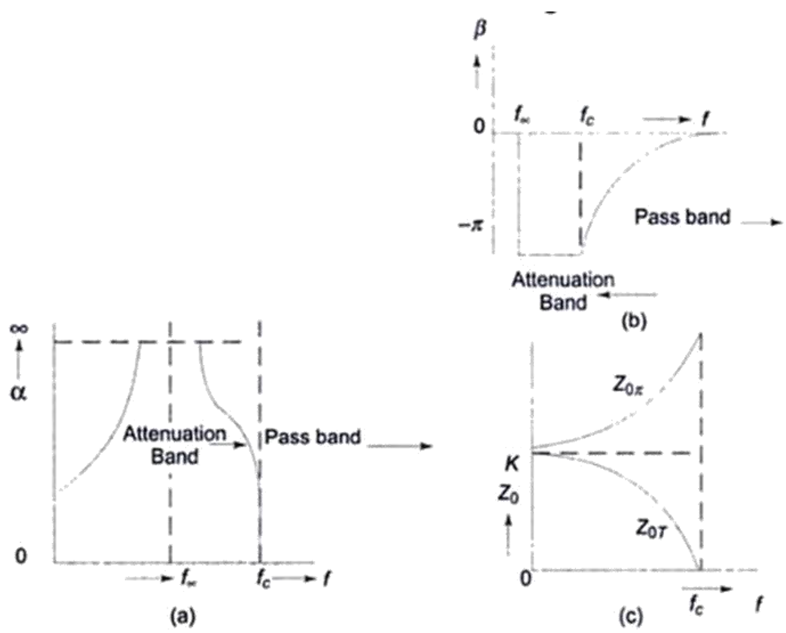


the m-derived \[\pi\]-section, the resonant circuit is constituted by the series arm inductance and capacitance

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m} C}$$

$$\omega_r^2 = \omega_\alpha^2 = \frac{1-m^2}{4LC}$$

$$\omega_\alpha = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \; or \; f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$



Conclusion:

The two-port network and able to set up at two-port circuit and perform the calculation of parameters using the measured voltages and currents values. Thus, it is important for us to select the suitable apparatus and methods for the experiment and be careful and patient when undergoing the experiment and calculations so that the errors will be minimize. However, it is not necessary for the experimental results to be perfect matched to the theoretical results because it is impossible, as long the experimental results are in tolerance range, the results are considered as accurate.

Reference:

[1].Christopher K. Alexander and Matther N.O. Sadiku (2016)
[2]. Paranjothi SR, "Electric Circuits Analysis," New Age International Ltd., New Delhi, 1996.
[3].Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi,
[4]. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education 2004

Post Test MCQs:

1.The two port networks are connected in cascade, the combination is to be represented as a single two-Port network. The parameters of the network are obtained by multiplying the individual matrix

a. z-parameter
b. h-parameter
c. y-parameter
d. ABCD parameter

2. Two port Z parameter not exist for the circuit if

a. $\Delta z = 0$
b. Δz
c. $\Delta z = 1$
d. always exist

3. The pass band of a constant *k* filter with Z_1 and Z_2 as series and shunt arm impedances is given by

a. . $-1 < \frac{Z_1}{4Z_2} < 0$

b . $-2 < \frac{Z_1}{4Z_2} < -1$

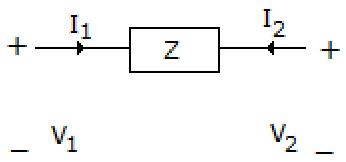
c. $1 < \frac{Z_1}{4Z_2} < 0$

d. $0 < \frac{Z_1}{4Z_2} < 1$

4.
A constant *k* high pass filter has f_c = 3000 Hz. At f = 1000 Hz, the phase shift is

a. Zero
b. .π
c. 2π
d. More than π

5. Which one of the following parameters does not exist for the two port network shown in the given figure?



a. ABCD
b. Z
c. h
d. y

UNIT – IV
DC TRANSIENT ANALYSIS

Aim

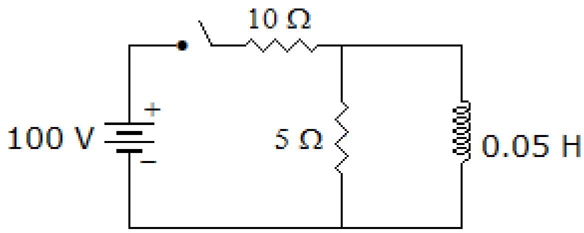
To analyze the behavior of the circuit's response in frequency domain.

Pre-Requisites:

Knowledge of Basic Mathematics – II & Basic Electronics Engineering

Pre - MCQs:

1.In the circuit of figure the current through 5 Ω resistance at *t* = 0⁺ is



a. 0 A
b. 10 A
C.. 6.67 A
c. 5.1 A

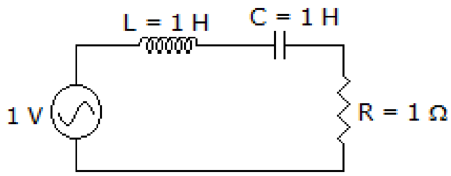
Answer: Option **C**

Explanation:

Current through inductance is zero. Current through 5 ohm resistance = 6.67A.

2. A variable resistance R and capacitive reactance X_C are fed by an voltage. The current locus is
a. a semi circle in 4th quadrant
b. .a semi circle in first quadrant
c. a straight line in 4th quadrant
d. a straight line first quadrant

3.
The damping coefficient for the given circuit is ___________



a..1.2
b.1.4
c.3
d.2

Introduction:

In this chapter we shall study transient response of the RL, RC series and RLC circuits with external DC excitations. Transients are generated in Electrical circuits due to abrupt changes in the operating conditions when energy storage elements like Inductors or capacitors are present. Transient response is the dynamic response during the initial phase before the steady state response is achieved when such abrupt changes are applied. To obtain the transient response of such circuits we have to solve the differential equations which are the governing equations representing the electrical behavior of the circuit. A circuit having a single energy storage element i.e. either a capacitor or an Inductor is called a Single order circuit and it's governing equation is called a First order Differential Equation. A circuit having both Inductor and a Capacitor is called a Second order Circuit and it's governing equation is called a Second order Differential Equation. The variables in these Differential Equations are currents and voltages in the circuitas a function of time.

A solution is said to be obtained to these equations when we have found an expression for the dependent variable that satisfies both the differential equation and the prescribed initial conditions. The solution of the differential equation represents the Response of the circuit. Now we will find out the response of the basic RL and RC circuits with DC Excitation.

RL CIRCUIT with external DC excitation:
Let us take a simple RL network subjected to external DC excitation as shown in the figure. The circuit consists of a battery whose voltage is *V* in series with a switch, a resistor *R*, and an inductor *L*. The switch is closed at $t = 0$.

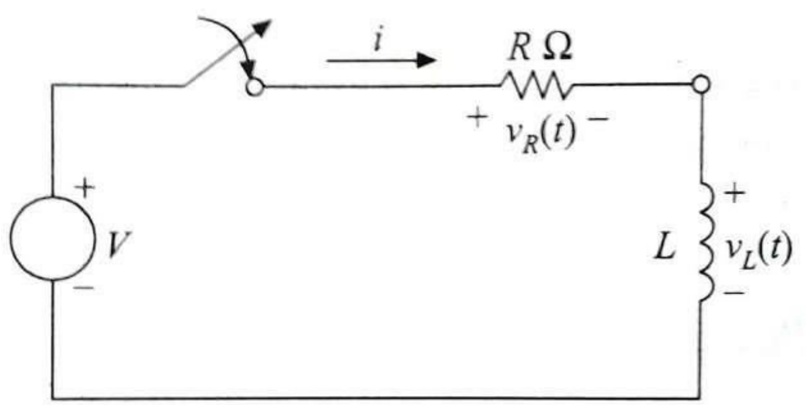


Fig: RL Circuit with external DC excitation

When the switch is closed current tries to change in the inductor and hence a voltage $V_L(t)$ is induced across the terminals of the Inductor in opposition to the applied voltage. The rate of change of current decreases with time which allows current to build up to it's maximum value.

It is evident that the current $i(t)$ is zero before $t = 0$.and we have to find out current $i(t)$for time t >0. We will find $i(t)$for time t >0 by writing the appropriate circuit equation and then solving it by separation of the variables and integration.
Applying Kirchhoff's voltage law to the above circuit we get :

$$V = v_R(t) + v_L(t)$$

$$i(t) = 0 \text{ for } t < 0 \text{ and}$$

Using the standard relationships of Voltage and Current for the Resistors and Inductors we can rewrite the above equations as

$$V = Ri + L\,di/dt \text{ for } t > 0$$

One direct method of solving such a differential equation consists of writing the equation in such a way that the variables are separated, and then integrating each side of the equation. The variables in the above equation are I and t. This equation is multiplied by dt and arranged with the variables separated as shown below:

$$Ri.\,dt + L\,di = V.\,dt$$

$$\text{i.e } L\,di = (V - Ri)dt$$

$$\text{i.e } L\,di \,/\, (V - Ri) = dt$$

Next each side is integrated directly to get :

$$-(L/R)\ln(V - Ri) = t + k$$

Where k is the integration constant. In order to evaluate k, an initial condition must be invoked. Prior to t = 0, i (t) is zero, and thus i (0−) = 0. Since the current in an inductor can not change by a finite amount in zero time with out being associated with an infinite voltage, we have i (0+) = 0. Setting i = 0 at t = 0, in the above equation we obtain

$$-(L/R)\ln(V) = k$$

and, hence,

$$-L/R[\ln(V - Ri) - \ln V] = t$$

Rearranging we get

$$\ln[(V - Ri)/V] = -(R/L)t$$

Taking antilogarithm on both sides we get

$$(V - Ri)/V = e^{-Rt/L}$$

From which we can see that

$$i(t) = (V/R) - (V/R)e^{-Rt/L} \text{ for } t > 0$$

Thus, an expression for the response valid for all time twould be

$$i(t) = V/R\,[1 - e^{-Rt/L}]$$

This is normally written as:

$$i(t) = V/R\,[1 - e^{-t/\tau}]$$

where 'τ' is called the *time constant* of the circuit and it's unit is seconds.

The voltage across the *resistance* and the *Inductor* for t >0can be written as :

$$v_R(t) = i(t).R = V\,[1 - e^{-t/\tau}]$$

$$v_L(t) = V - v_R(t) = V - V\,[1 - e^{-t/\tau}] = V(e^{-t/\tau})$$

A plot of the current i(t) and the voltages $v_R(t)$ & $v_L(t)$ is shown in the figure below.

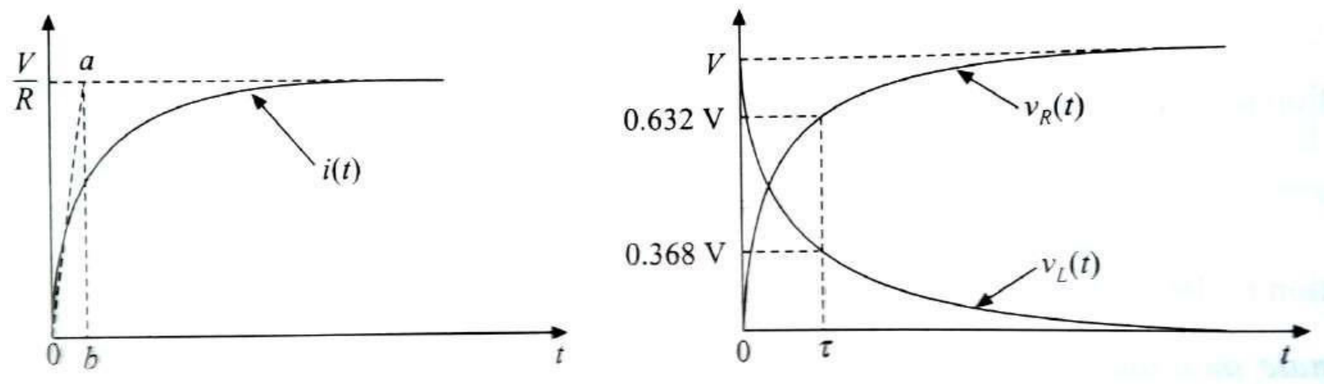


Fig: Transient current and voltages in the Series RL circuit.

At t = 'τ' the voltage across the inductor will be

$$v_L(\tau) = V\,(e^{-\tau/\tau}) = V/e = 0.36788\,V$$

and the voltage across the Resistor will be $\quad v_R(\tau) = V\,[1 - e^{-\tau/\tau}] = 0.63212\ V$

The plots of current i(t) and the voltage across the Resistor $v_R(t)$ are called *exponential growth* curves and the voltage across the inductor $v_L(t)$ is called *exponential decay* curve.

RCCIRCUIT with external DC excitation:

A series RC circuit with external DC excitation V volts connected through a switch is shown in the figure below. If the capacitor is not charged initially i.e. it's voltage is zero ,then after the switch S is closed at time t=0, the capacitor voltage builds up gradually and reaches it's steady state value of V volts after a finite time. The charging current will be maximum initially (since initially capacitor voltage is zero and voltage across a capacitor cannot change instantaneously) and then it will gradually comedown as the capacitor voltage starts building up. The current and the voltage during such charging periods are called Transient Current and Transient Voltage.

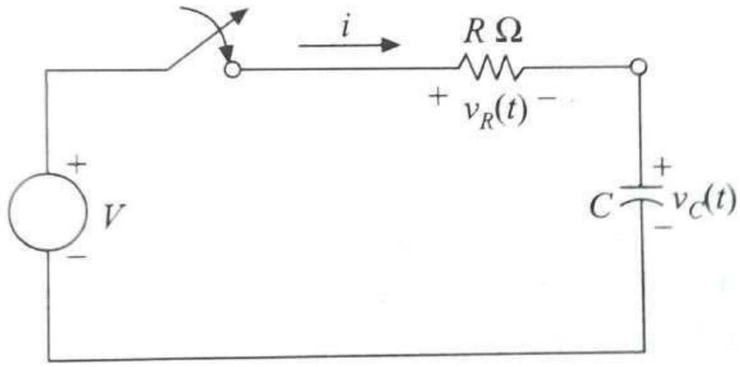


Fig: RC Circuit with external DC excitation

Applying KVL around the loop in the above circuit we can write

$$V = v_R(t) + v_C(t)$$

Using the standard relationships of voltage and current for an Ideal Capacitor we get

$$v_C(t) = (1/C)\int i(t)dt \quad \text{or } i(t) = C.[dv_C(t)/dt]$$

and using this relation, $v_R(t)$ can be written as $v_R(t) = Ri(t) = R.C.[dv_C(t)/dt]$

Using the above two expressions for $v_R(t)$ and $v_C(t)$ the above expression for V can be rewritten as :

$$V = R.C.[dv_C(t)/dt] + v_C(t)$$

$$\text{Or finally } dv_C(t)/dt + (1/RC).v_C(t) = V/RC$$

The inverse coefficient of $v_C(t)$ is known as the time constant of the circuit τ and is given by τ = RC and it's units are seconds.

The above equation is a first order differential equation and can be solved by using the same method of *separation of variables* as we adopted for the LC circuit.

Multiplying the above equation $dv_C(t)/dt + (1/RC).v_C(t) = V/RC$

both sides by 'dt' and rearranging the terms so as to separate the variables $v_C(t)$ and t we get:

$$dv_C(t) + (1/RC).v_C(t).dt = (V/RC).dt$$

$$dv_C(t) = [(V/RC) - (1/RC).v_C(t)].dt$$

$$dv_C(t) / [(V/RC) - (1/RC).v_C(t)] = dt$$

$$R.C.dv_C(t)/[(V - v_C(t)] = dt$$

Now integrating both sides w.r.t their variables i.e. '$v_C(t)$' on the LHS and't' on the RHS we get

$$-RC \ln [V - v_C(t)] = t + k$$

where 'k' is the constant of integration. In order to evaluate k, an initial condition must be invoked. Prior to $t = 0$, $v_C(t)$ is zero, and thus $v_C(t)(0-) = 0$. Since the voltage across a capacitor cannot change by a finite amount in zero time, we have $v_C(t)(0+) = 0$. Setting $v_C(t) = 0$ at $t = 0$, in the above equation we obtain:

$$-RC \ln [V] = k$$

and substituting this value of $k = -RC \ln [V]$ in the above simplified equation $-RC \ln [V - v_C(t)] = t + k$ we get :

$$-RC \ln [V - v_C(t)] = t - RC \ln [V]$$

i.e. $-RC \ln [V - v_C(t)] + RC \ln [V] = t$ i.e. $-RC [\ln \{V - v_C(t)\} - \ln (V)] = t$

i.e. $[\ln \{V - v_C(t)\}] - \ln [V]\} = -t/RC$

i.e. $\ln [\{V - v_C(t)\}/(V)] = -t/RC$

Taking anti logarithm we get $[\{V - v_C(t)\}/(V)] = e^{-t/RC}$

i.e $v_C(t) = V(1 - e^{-t/RC})$

which is the voltage across the capacitor as a function of time .

The voltage across the Resistor is given by : $v_R(t) = V - v_C(t) = V - V(1 - e^{-t/RC}) = V.e^{-t/RC}$

And the current through the circuit is given by: $i(t) = C.[dv_C(t)/dt] = (CV/CR)e^{-t/RC} = (V/R)e^{-t/RC}$

Or the othe other way: $i(t) = v_R(t)/R = (V.e^{-t/RC})/R = (V/R)e^{-t/RC}$

In terms of the time constant τ the expressions for $v_C(t)$, $v_R(t)$ and $i(t)$ are given by :

$$v_C(t) = V(1 - e^{-t/RC})$$

$$v_R(t) = V.e^{-t/RC}$$

$$i(t) = (V/R)e^{-t/RC}$$

The plots of current $i(t)$ and the voltages across the resistor $v_R(t)$ and capacitor $v_C(t)$ are shown in the figure below.

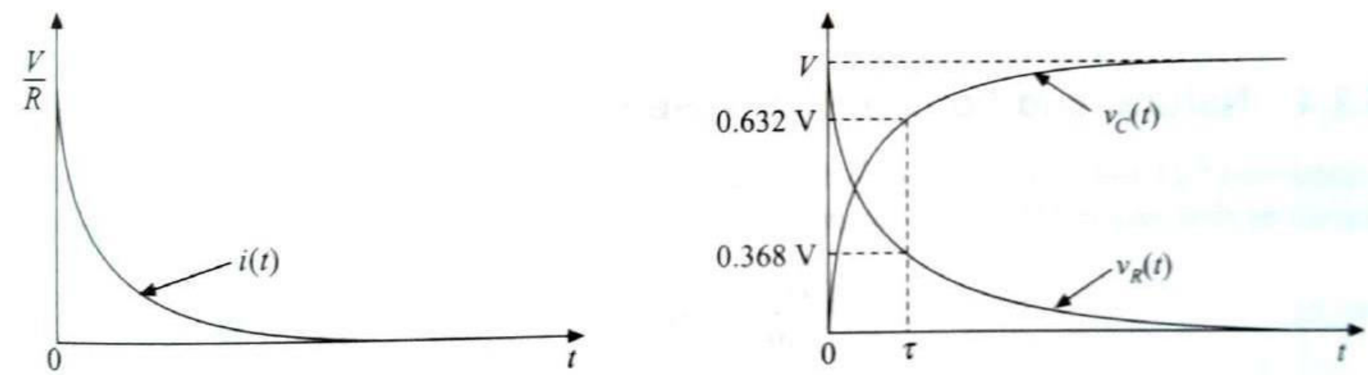


Fig : Transient current and voltages in RC circuit with DC excitation.

At t = 'τ' the voltage across the capacitor will be:

$$v_C(\tau) = V\,[1 - e^{-\tau/\tau}] = 0.63212\ V$$

the voltage across the Resistor will be:

$$v_R(\tau) = V\,(e^{-\tau/\tau}) = V/e = 0.36788\ V$$

and the current through the circuit will be:

$$i(\tau) = (V/R)\,(e^{-\tau/\tau}) = V/R.\,e = 0.36788\ (V/R)$$

Thus it can be seen that after one time constant the charging current has decayed to approximately 36.8% of it's value at t=0 . At t= 5 τ charging current will be

$$i(5\tau) = (V/R)\,(e^{-5\tau/\tau}) = V/R.\,e^{5} = 0.0067(V/R)$$

This value is very small compared to the maximum value of (V/R) at t=0. Thus it can be assumed that the capacitor is fully charged after 5 time constants.

The following similarities may be noted between the equations for the transients in the LC and RC circuits:

- The transient voltage across the Inductor in a LC circuit and the transient current in the RC circuit have the same form $k.(e^{-t/\tau})$
- The transient current in a LC circuit and the transient voltage across the capacitor in the RC circuit have the same form $k.(1-e^{-t/\tau})$

But the main difference between the RC and RL circuits is the effect of resistance on the duration of the transients.

- In a RL circuit a large resistance shortens the transient since the time constant $\tau = L/R$ Becomes small.
- Where as in a RC circuit a large resistance prolongs the transient since the time constant $\tau = RC$ becomes large.

Discharge transients: Consider the circuit shown in the figure below where the switch allows both charging and discharging the capacitor. When the switch is position 1 the capacitor gets charged to the applied voltage V. When the switch is brought to position 2, the current discharges from the positive terminal of the capacitor to the negative terminal through the resistor R as shown in the figure (b). The circuit in position 2 is also called *source free circuit* since there is no any applied voltage.

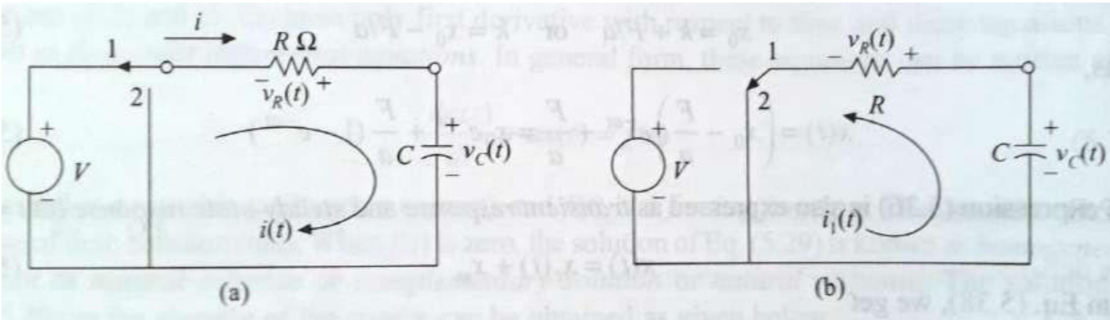


Fig: RC circuit (a) During Charging (b) During Discharging

The current i_1 flow is in opposite direction as compared to the flow of the original charging current i. This process is called the *discharging of the capacitor*.The decaying voltage and the current are called the *discharge transients*.The resistor ,during the discharge will oppose the flow of current with the polarity of voltage as shown. Since there is no any external voltage source ,the algebraic sum of the voltages across the Resistance and the capacitor will be zero (applying KVL) .The resulting loop equation during the discharge can be written as

$$v_R(t)+v_C(t) = 0 \quad \text{or } v_R(t) = - v_C(t)$$

We know that $v_R(t) = R.i(t) = R. C.dv_C(t) /dt$. Substituting this in the first loop equation we get

$$R. C.dv_C(t)/dt + v_C(t) = 0$$

The solution for this equation is given by $v_C(t) = Ke^{-t/\tau}$ where K is a constant decided by the initial conditions and τ =RC is the time constant of the RC circuit

The value of K is found out by invoking the initial condition $v_C(t) = V$ @t = 0

Then we get K = V and hence $v_C(t) = Ve^{-t/\tau}$; $v_R(t) = -Ve^{-t/\tau}$ and $i(t) = v_R(t)/R = (-V/R)e^{-t/\tau}$

The plots of the voltages across the Resistor and the Capacitor are shown in the figure below.

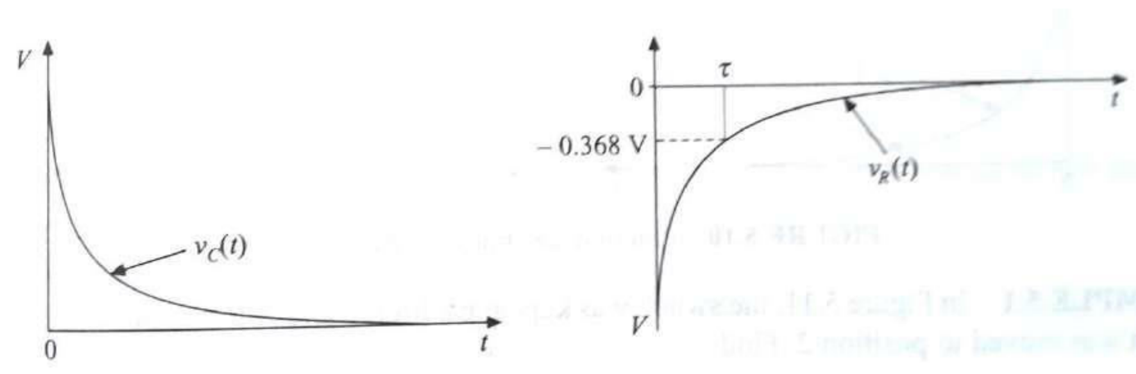


Fig: Plot of Discharge transients in RC circuit

Decay transients: Consider the circuit shown in the figure below where the switch allows both growing and decaying of current through the Inductance . When the switch is position 1 the current through the Inductance builds up to the steady state value of V/R. When the switch is brought to position 2, the current decays gradually from V/R to zero. The circuit in position 2 is also called a *source free circuit*since there is no any applied voltage.

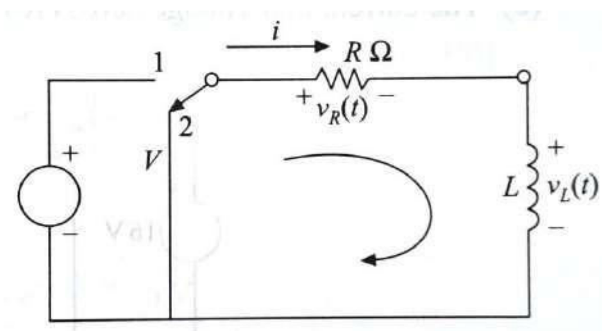


Fig: Decay Transient In RL circuit

The current flow during decay is in the same direction as compared to the flow of the original growing /build up current. The decaying voltage across the Resistor and the current are called the *decay transients*.. Since there is no any external voltage source ,the algebraic sum of the voltages across the Resistance and the Inductor will be zero (applying KVL) .The resulting loop equation during the discharge can be written as

$$v_R(t)+v_L(t) = R.i(t)+L.di(t)/dt=0 \quad \text{and } v_R(t) = - v_L(t)$$

The solution for this equation is given by $i(t) = Ke\text{-}^{t/\tau}$ where K is a constant decided by the initial conditions and τ =L/R is the time constant of the RL circuit.

The value of the constant K is found out by invoking the initial condition i(t) = V/R @t = 0 Then

we get K = V/R and hence $i(t) = (V/R).e^{-t/\tau}$; $v_R(t) = R.i(t) = Ve^{-t/\tau}$ and $v_L(t) = - Ve^{-t/\tau}$

The plots of the voltages across the Resistor and the Inductor and the decaying current through the circuit are shown in the figure below.

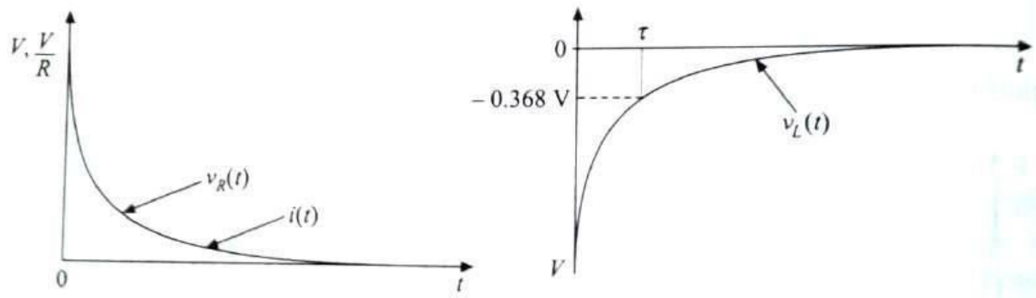


Fig: Plot of Decay transients in RL circuit

The Concept of Natural Response and forced response:

The RL and RC circuits we have studied are with external DC excitation. These circuits without the external DC excitation are called *source free circuits* and their Response obtained by solving the corresponding differential equations is known by many names. Since this response depends on the general *nature* of the circuit (type of elements, their size, their interconnection method etc.,) it is often called a *Natural response*. However any real circuit we construct cannot store energy forever. The resistances intrinsically associated with Inductances and Capacitors will eventually dissipate the stored energy into heat. The response eventually dies down,. Hence it is also called Transient response. As per the mathematician's nomenclature the solution of such a homogeneous linear differential equation is called Complementary function.

When we consider independent sources acting on a circuit, part of the response will resemble the nature of the particular source. (Or forcing function) This part of the response is called particular solution. , the steady state response or forced response. This will be complemented by the complementary function produced in the source free circuit. The complete response of the circuit is given by the sum of the complementary function and the particular solution. In other words:

The Complete response = Natural response + Forced response

There is also an excellent mathematical reason for considering the complete response to be composed of two parts—the *forced response* and the *natural response*. The reason is based on the fact that the solution of any linear differential equation may be expressed as the sum of two parts: the *complementary solution*(natural response) and the *particular solution*(forced response).

Determination of the Complete Response:

Let us use the same RL series circuit with external DC excitation to illustrate how to determine the complete response by the addition of the natural and forced responses. The circuit shown in the figure

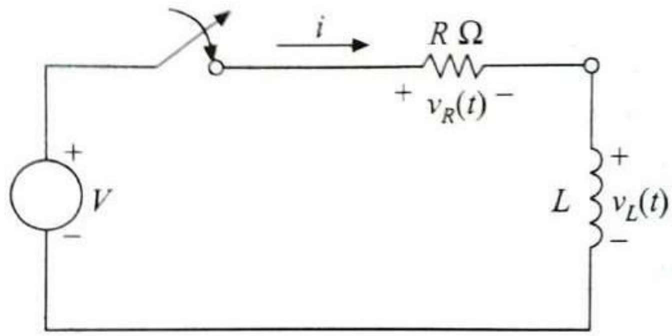


Fig: RL circuit with external DC excitation

was analyzed earlier, but by a different method. The desired response is the current *i (t)*, and now we first express this current as the sum of the natural and the forced current,

$$i = i_n + i_f$$

The functional form of the natural response must be the same as that obtained without any sources. We therefore replace the step-voltage source by a short circuit and call it the *RL source free* series loop. And i_n can be shown to be :

$$i_n = Ae^{-Rt/L}$$

where the amplitude *A* is yet to be determined; since the initial condition applies to the *complete* response, we cannot simply assume $A = i(0)$. We next consider the forced response. In this particular problem the forced response is constant, because the source is a constant *V* for all positive values of time. After the natural response has died out, there can be no voltage across the inductor; hence the all ythe applied voltage *V* appears across *R*, and the forced response is simply

$$i_f = V/R$$

Note that the forced response is determined completely. There is no unknown amplitude. We next combine the two responses to obtain :

$$i = Ae^{-Rt/L} + V/R$$

And now we have to apply the initial condition to evaluate *A*. The current is zero prior to $t = 0$, and it cannot change value instantaneously since it is the current flowing through an inductor. Thus, the current is zero immediately after $t = 0$, and

$$A + V/R = 0$$

Sothat

$$A = -V/R$$

And $\quad i = (V/R)(1 - e^{-Rt/L})$

Note carefully that *A* is not the initial value of i, since $A = -V/R$, while $i(0) = 0$.

But In source-free circuits, A would be the initial value of the response given by $i_n = I_0 e^{-Rt/L}$ (where I_0 =A is the current at time t=0). When forcing functions are present, however, we must first find the initial value of the complete response and then substitute this in the equation for the complete response to find A. Then this value of A is substituted in the expression for the total response i

A more general solution approach:

The method of solving the differential equation by separating the variables or by evaluating the complete response as explained above may not be possible always. In such cases we will rely on a verypowerful method, the success of which will depend upon our intuition or experience. We simply guess or assume a form for the solution and then test our assumptions, first by substitution in the differential equation, and then by applying the given initial conditions. Since we cannot be expected to guess the exact numerical expression for the solution, we will assume a solution containing several unknown constants and select the values for these constants in order to satisfy the differential equation and the initialconditions.
Many of the differential equations encountered in circuit analysis have a solution which may be represented by the exponential function or by the sum of several exponential functions. Hence Let us assume a solution for the following equation corresponding to a source free RL circuit

$$[\ di/dt + (R\ i/L)] = 0$$

inexponentialformas

$$i\ (t) = A.e^{s1t}$$

where A ands1 are constants to be determined. Now substituting this assumed solution in the original governing equation wehave:

$$A\ .\ s1\ .\ e^{s1t} + A\ .e^{s1t}\ .\ R/L = 0$$

Or

$$(s1 + R/L).\ A.e^{s1t} = 0$$

In order to satisfy this equation for all values of time, it is necessary that A = 0, or s1 = −∞, or s1 = −R/L. Butif A= 0ors1=−∞,then every responseiszero;neithercanbeasolutiontoourproblem.Therefore, we must choose

$$s1 = -R/L$$

And our assumed solution takes on the form:

$$i\ (t) = A.e^{-Rt/L}$$

The remaining constant must be evaluated by applying the initial condition $i\ (0) = I_0$. Thus, $A = I_0$, and the final form of the assumed solution is(again):

$$i\ (t) = I_0.e^{-Rt/L}$$

A Direct Route: The Characteristic Equation:

In fact, there is a more direct route that we can take. To obtain the solution for the first order DE we solveds1 + R/L= 0 which is known as the *characteristic equation* and then substituting this value of *s1*=- R/Lin the assumed solution $i\ (t) = A.e^{s1t}$ which is same in this direct method also. We can obtain the characteristic equation directly from the differential equation, without the need for substitution of our trial solution. Consider the general first-order differential equation:

$$a(d\ f/dt) + bf = 0$$

where a andbare constants. We substitute s for the differentiation operator d/dt in theoriginal differential equation resulting in

$$a(d f/dt) + bf = (as + b) f = 0$$

From this we may directly obtain the characteristic equation: $as + b = 0$

which has the single root $s = -b/a$. Hence the solution to our differential equation is then given by :

$$f = A.e^{-bt/a}$$

This basic procedure can be easily extended to second-order differential equations which we will encounter for RLC circuits and we will find it useful since adopting the variable separation method is quite complex for solving second order differential equations.

RLC CIRCUITS:

Earlier, we studied circuits which contained only one energy storage element, combined with a passive network which partly determined how long it took either the capacitor or the inductor to charge/discharge. The differential equations which resulted from analysis were always first-order. In this chapter, we consider more complex circuits which contain both an inductor and a capacitor. The result is a second-order differential equation for any voltage or current of interest. What we learned earlier is easily extended to the study of these so-called *RLC* circuits, although now we need two initial conditions to solve each differential equation. There are two types of RLC circuits: *Parallel RLC circuits* and *Series circuits* .Such circuits occur routinely in a wide variety of applications and are very important and hence we will study both these circuits.

Parallel RLC circuit:

Let us first consider the simple parallel RLC circuit with DC excitation as shown in the figure below.

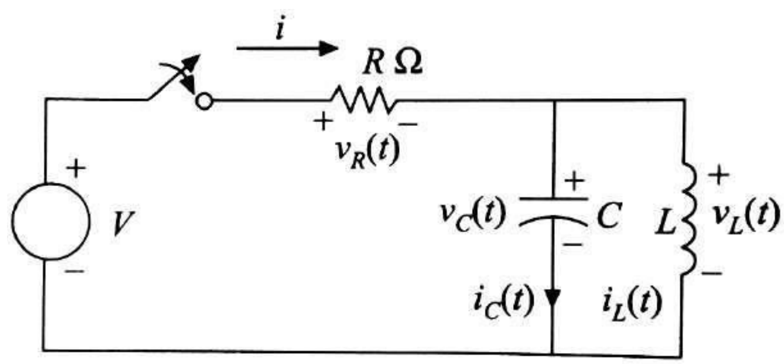


Fig:Parallel *RLC* circuit with DC excitation.

For the sake of simplifying the process of finding the response we shall also assume that the initial current in the inductor and the voltage across the capacitor are zero. Then applying the Kirchhoff's current law (KCL)($i = i_C + i_L$)to the common node we get the following integral differential equation:

$$(V-v)/R = 1/L\int_{t_0}^{t} v\,dt' + C.dv/dt$$

$$V/R = v/R + 1/L\int_{t_0}^{t} v\,dt' + C.dv/dt$$

Where v = $v_C(t) = v_L(t)$ is the variable whose value is to be obtained .
When we differentiate both sides of the above equation once with respect to time we get the standard Linear second-order homogeneous differential equation

$$C.(d^2v/dt^2)+(1/R).(dv/dt)+(1/L).v=0 \; (d^2v/dt^2)+$$

$$(1/RC).(dv/dt)+(1/LC).v=0$$

whose solution v(t) is the desired response. This

can be written in the form:

$$[s^2 + (1/RC)s + (1/LC)].v(t) = 0$$

where 's' is an operator equivalent to (d/dt) and the corresponding *characteristic equation* (as explained earlier as a direct route to obtain the solution) is then given by :

$$[s^2 + (1/RC)s + (1/LC)] = 0$$

This equation is usually called the *auxiliary equation* or the *characteristic equation,* as we discussed earlier .If it can be satisfied, then our assume dsolution is correct. This is a quadratic equation and the roots s_1 and s_2 are given as :

$$s_1 = -1/2RC + \sqrt{[(1/2RC)^2 - 1/LC]} \quad s_2 =$$
$$-1/2RC - \sqrt{[(1/2RC)^2 - 1/LC]}$$

And we have the general form of the response as :

$$v(t) = A_1e^{s1t} + A_2e^{s2t}$$

where s_1 and s_2 are given by the above equations and A_1 and A_2 are two arbitrary constants which are to be selected to satisfy the two specified initial conditions.

Definition of Frequency Terms:

The form of the natural response as given above gives very little insight into the nature of the curve we might obtain if *v(t)* were plotted as a function of time. The relative amplitudes of $A1$ and $A2$, for example, will certainly be important in determining the shape of the response curve. Further the constants $s1$ and $s2$ can be real numbers or conjugate complex numbers, depending upon the values of R, L, and C in the given network. These two cases will produce fundamentally different response forms. Therefore, it will be helpful to make some simplifying substitutions in the equations for $s1$ and $s2$.Since the exponents $s1t$ and $s2t$ must be dimensionless, $s1$ and $s2$ must have the unit of some dimensionless quantity "per second." Hence in the equations for $s1$ and $s2$ we see that the units of $1/2RC$ and $1/\sqrt{LC}$ must also be s^{-1}(i.e., seconds^{-1}). Units of this type are called *frequencies.*
Now two new terms are defined as below :

$$\omega 0 = 1/\sqrt{LC}$$

whichis termedas *resonantfrequency* and

$$\alpha = 1/2RC$$

which is termed asthe *exponential damping coefficient*

α the exponential damping coefficient is a measure of how rapidly the natural response decays or damps out to its steady, final value(usually zero). And s, s_1, and s_2, are called *complex frequencies.*
We should note that s_1, s_2, α, and ω_0 are merely symbols used to simplify the discussion of *RLC* circuits. They are not mysterious new parameters of any kind. It is easier, for example, to say "*alpha*" than it is to say "*the reciprocal of 2RC.*"

Now we can summarize these results.

The response of the parallel *RLC* circuit is given by :

$$v(t)=A_1e^{s1t}+A_2e^{s2t} \cdots\cdots\cdots [1]$$

where

$$s_1=-\alpha+\sqrt{\alpha^2-\omega_0{}^2} \cdots\cdots [2]$$

$$s_2=-\alpha-\sqrt{\alpha^2-\omega_0{}^2} \cdots\cdots [3]$$

$$\alpha = 1/2RC \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [4]$$

and

$$\omega 0=1/\sqrt{LC} \ldots\ldots\ldots [5]$$

A_1 and A_2must be found by applying the given initial conditions.
We note three basic scenarios possible with the equations for s1 and s2 depending on the relative values of α and ω_0 (which are in turn dictated by the values of R, L, and C).

Case A:

$\alpha > \omega 0$,i.e when $(1/2RC)^2>1/LC$ s_1 and s_2 will both be negative real numbers, leading to what is referred to as an *over damped response* given by :

$$v(t) = A_1e^{s1t}+ A_2e^{s2t}$$

Since s_1 and $s_2$are both negative real numbers this is the (algebraic) sum of two decreasing exponential terms. Sinces2isalargernegativenumberitdecaysfasterandthentheresponseisdictatedbythefirst term A_1e^{s1t}.

Case B :

$\alpha = \omega_0$, ,i.e when $(1/2RC)^2=1/LC$, s_1 and $s_2$are equal which leads to what is called a *critically damped response* given by :

$$v(t) = e^{-\alpha t}(A_1t + A_2)$$

Case C :

$\alpha < \omega 0$,i.e when $(1/2RC)^2 < 1/LC$both s_1 and s_2 will have nonzero imaginary components, leading to what is known as an *under damped response* given by :

$$v(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

where $\omega_d$is called *natural resonant frequency* and is given given by:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

We should also note that the general response given bythe above equations [1] through [5] describe not only the voltage but all three branch currents in the parallel *RLC* circuit; the constants A_1 and A_2 will be different for each, of course.

Transient response of a series RLC circuit:

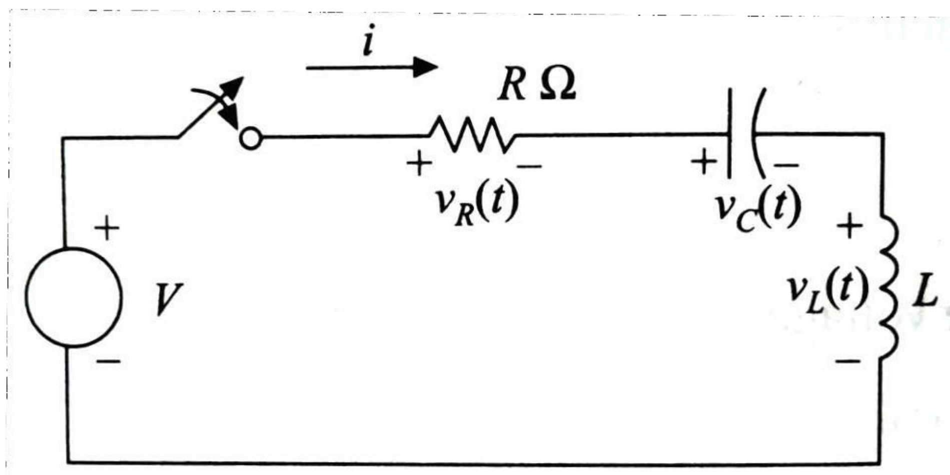


Fig: Series RLC circuit with external DC Excitation

Applying KVL to the series RLC circuit shown in the figure above at t= 0 gives the following basic relation :

$$V = v_R(t) + v_C(t) + v_L(t)$$

Representing the above voltages in terms of the current iin the circuit we get the following integral differential equation:

$$Ri + 1/C\int idt + L. (di/dt)= V$$

To convert it into a differential equation it is differentiated on both sides with respect to time and we get

$$L(d^2i/dt^2)+ R(di/dt)+ (1/C)i = 0$$

This can be written in the form

$[S^2 + (R/L)s + (1/LC)].i = 0$ where 's' is an operator equivalent to (d/dt)

And the corresponding characteristic equation is then given by

$$[s^2 + (R/L)s + (1/LC)] = 0$$

This is in the standard quadratic equation form and the roots s_1 and s_2 are given by

$$s_1, s_2 = -R/2L \pm \sqrt{[(R/2L)^2 - (1/LC)]} = -\alpha \pm \sqrt{(\alpha^2 - \omega_0^2)}$$

Where α is known as the same *exponential damping coefficient t*and ω_0 is known as the same *Resonant frequency* as explained in the case of Parallel RLC circuit and are given by :

$$\alpha = R/2L \quad \text{and} \quad \omega_0 = 1/\sqrt{LC}$$

and A_1 and A_2 must be found by applying the given initial conditions.

Here also we note three basic scenarios with the equations for s1 and s2 depending on the relative sizes of α and ω_0 (dictated by the values of R, L, and C).

Case A:
$\alpha > \omega_0$,i.e when $(R/2L)^2 > 1/LC$, s_1 and s_2 will both be negative real numbers, leading to what is referred to as an *over damped response* given by :

$$i(t) = A_1e^{s1t} + A_2e^{s2t}$$

Since s_1 and s_2 are both be negative real numbers this is the (algebraic) sum of two decreasing exponential terms. Sinc s2 is a larger negative number it decays faster and then the response is dictated by the first term A_1e^{s1t}.

Case B :
$\alpha = \omega_0$, ,i.e when $(R/2L)^2 = 1/LC$ s_1 and s_2 are equal which leads to what is called a *critically damped response* given by :

$$i(t) = e^{-\alpha t}(A_1t + A_2)$$

Case C :
$\alpha < \omega_0$,i.e when $(R/2L)^2 < 1/LC$ both s_1 and s_2 will have nonzero imaginary components, leading to what is known as an *under damped response* given by :

$$i(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

where ω_d is called *natural resonant frequency* and is given given by:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Here the constants A_1 and A_2 have to be calculated out based on the initial conditions case by case.

Summary of the Solution Process:

In summary, then, whenever we wish to determine the transient behavior of asimple three-element RLCcircuit, we must first decide whether it is a series or a parallel circuit, so that we may use the correct relationshipfor α. The two equations are

$\alpha = 1/2RC$ (parallel RLC)

$\alpha = R/2L$ (series RLC)

Our second decision is made after comparing α with ω_0, which is given for either circuit by

$\omega_0 = 1/\sqrt{LC}$

- If $\alpha > \omega_0$, the circuit is *over damped*, and the natural response has the form

$$f_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

where

$$s_{1,2} = -\alpha \pm \sqrt{(\alpha^2 - \omega_0^2)}$$

- If $\alpha = \omega_0$, then the circuit is *critically damped* and

$$f_n(t) = e^{-\alpha t}(A1t + A2)$$

- And finally, if $\alpha < \omega_0$, then we are faced with the *underdamped* response,

$$f_n(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

where

$$\omega_d = \sqrt{(\omega_0^2 - \alpha^2)}$$

Solution using Laplace transformation method:

In this topic we will study Laplace transformation method of finding solution for the differential equations that govern the circuit behavior. This method involves three steps:

- First the given Differential equation is converted into "s" domain by taking it's Laplace transform and an algebraic expression is obtained for the desired variable
- The transformed equation is split into separate terms by using the method of Partial fraction expansion
- Inverse Laplace transform is taken for all the individual terms using the standard inverse transforms.

The expression we get for the variable in time domain is the required solution.

For the ease of reference a table of important transform pairs we use frequently is given below.

Table of Important Transform pairs

$f(t)$ (*Function*)	$F(s)$ (*Laplace Transform*)
$u(t)$ (unit step)	$1/s$
$\delta(t)$ (unit impulse)	1
e^{-at}	$\frac{1}{(s+a)}$
$\sin \omega t$	$\frac{w}{(s^2+\omega^2)}$
$\cos \omega t$	$\frac{s}{(s^2+\omega^2)}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{(s+a)}{(s+a)^2+\omega^2}$
t	$1/s^2$
$\frac{df(t)}{dt}$	$sF(s)$
$\int f(t)dt$	$F(s)/s$

This method is relatively simpler compared to Solving the Differential equations especially for higher order differential equations since we need to handle only algebraic equations in 's' domain.
This method is illustrated below for the series RL,RC and RLC circuits.

Series RL circuit with DC excitation:
Let us take the *series RL* circuit with external DC excitation shown in the figure below.

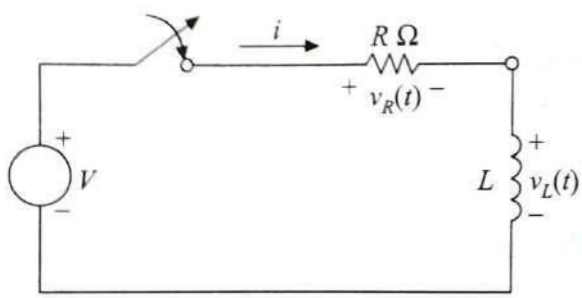


Fig: RL Circuit with external DC excitation

The governing equation is same as what we obtained earlier.

$$V = Ri + Ldi/dt \text{ for } t > 0$$

Taking Laplace transform of the above equation using the standard transform functions we get

$$V/s = R.I(s) + L[sI(s) - i(0)]$$

It may be noted here that i(0) is the initial value of the current at t=0 and since in our case at t=0 just when the switch is closed it is zero , the above equation becomes:

$$V/s = R.I(s) + L[sI(s)] = I(s)[R + L.s]$$

Or $I(s) = \left[\frac{(\frac{V}{L})}{s(s+\frac{R}{L})}\right] = \frac{A}{s} + \frac{B}{[s+\frac{R}{L}]}$ (Expressing in the form of Partial fractions)

Where $A = \left[\frac{(\frac{V}{L})}{(s+\frac{R}{L})}\right]_{s=0} = V/R$ and $B = \left[\frac{(\frac{V}{L})}{s}\right]_{s=-(R/L)} = -V/R$

Now substituting these values of A and B in the expression for $I(s) = \frac{A}{s} + \frac{B}{[s+\frac{R}{L}]}$ we get

$$I(s) = \frac{V/R}{s} - \frac{V/R}{[s+\frac{R}{L}]}$$

Taking inverse transform of the above expression for I(s)using the standard transform pairs we get the solution for i(t) as

$$i(t) = (V/R) - (V/R).e^{-(R/L)t} = (V/R)(1 - e^{-(R/L)t})$$

Which is the same as what we got earlier by solving the governing differential equation directly.

RC Circuit with external DC excitation:

Let us now take the *series RC* circuit with external DC excitation shown in the figure below.

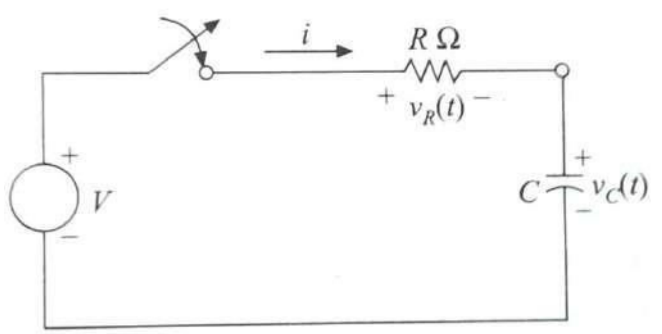


Fig: RC Circuit with external DC excitation

The governing equation is same as what we obtained earlier and is worked out again for easy understanding :

Applying KVL around the loop in the above circuit we can write:

$$V = v_R(t) + v_C(t)$$

Using the standard relationships of voltage and current for an Ideal Capacitor we get

$$v_C(t) = (1/C)\int i(t)dt \quad \text{or } i(t) = C.[dv_C(t)/dt]$$

(Assuming that the initial voltage across the capacitor $v_c(0) = 0$)

and using this relation, $v_R(t)$ can be written as $v_R(t) = Ri(t) = R.C.[d\, v_C(t)/dt]$

Using the above two expressions for $v_R(t)$ and $v_C(t)$the above expression for V can be rewritten as :

$$V = R.C.[d\, v_C(t)/dt] + v_C(t)$$

Now we will take Laplace transform of the above equation using the standard Transform pairs and rules:

$$V/s = R.C.s.v_C(s) + v_C(s)$$

$$V/s = v_C(s)\,(R.C.s.+ 1)$$

$$v_C(s) = (V/s)/(R.C.s + 1)$$

$$v_C(s) = (V/RC)/[s.(s + 1/RC)]$$

Now expanding this equation into partial fractions we get

$$v_C(s) = (V/RC)/[s.(s + 1/RC)] = A/s + B/(s + 1/RC) \text{ --------- } (1)$$

Where $A = (V/RC)/(1/RC)] = V$ and $B = (V/RC)/-(1/RC)] = -V$

Substituting these values of A and B into the above equation (1) for $v_C(s)$ we get

$$v_C(s) = (V/s) - [V/(s + 1/RC)] = V[(1/s) - \{1/(s + 1/RC)\}]$$

And now taking the inverse Laplace transform of the above equation we get

$$v_C(t) = V(1 - e^{-t/RC})$$

which is the voltage across the capacitor as a function of time and is the same as what we obtained earlier by directly solving the differential equation.

And the voltage across the Resistor is given by $v_R(t) = V - v_C(t) = V - V(1 - e^{-t/RC}) = V.e^{-t/RC}$

And the current through the circuit is given by $i(t) = C.[dv_C(t)/dt] = (CV/RC)e^{-t/RC} = (V/R)e^{-t/RC}$

Series RLC circuit with DC excitation:

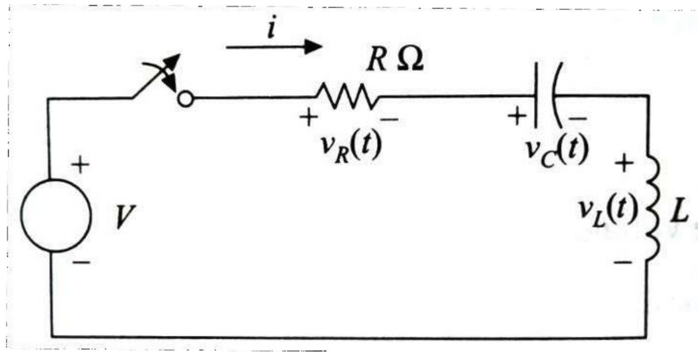


Fig: Series RLC circuit with DC excitation

The current through the circuit in the Laplace domain is given by :

$$I(s) = \frac{(V/s)}{(R + Ls + 1/Cs)}$$

[since L [V] = V/sand the Laplace equivalent of the series circuit is given by $Z(s) = (R + Ls + 1/Cs)$]

$$= V/ (Rs + Ls^2 + 1/C) = (V/L) / [s^2 + (R/L) s + 1/LC] = \frac{(V/L)}{(s+a)(s+b)}$$

Where the roots ‘a’ and ‘b’ are given by

$$a = -R/2L + \sqrt{(R/2L)^2 - 1/LC} \quad \text{and } b =$$

$$-R/2L - \sqrt{(R/2L)^2 - 1/LC}$$

It may be noted that there are three possible solutions for for I(s) and we will consider them.

Case A: Both aand b are real and not equal i.e. $(R/2L) > 1/\sqrt{LC}$

Then I(s) can be expressed as $I(s) = \frac{(V/L)}{(s+a)(s+b)} = \frac{K1}{(s+a)} + \frac{K2}{(s+b)}$

WhereK1 = $\left[\frac{(V/L)}{(s+b)} \right]_{s=-a} = \frac{(V/L)}{(b-a)}$

Where $K2 = [\frac{(V/L)}{(s+a)}]_{s=-b} = \frac{(V/L)}{(a-b)}$

Substituting these values of K1 and K2 in the expression for I(s) we get :

$I(s) = \frac{(V/L)}{(s+a)(s+b)} = \frac{(V/L)}{(b-a)}\frac{1}{(s+a)} + \frac{(V/L)}{(a-b)}\frac{1}{(s+b)}$ and

$i(t) = \frac{(V/L)}{(b-a)} e^{-at} + \frac{(V/L)}{(a-b)} e^{-bt}$

CaseB : Both a and b are real and equal i.e. (a=b=c) i.e. (R/2L) = 1/√ LC I(s)

$=(V/L)/(s+c)^2$ when a = b = c and

$i(t) = (V/L).\ t.\ e^{-ct}$

Case C : Both a and b are complexconjugates i.e.a = b* when (R/2L) < 1/√ LC

Adopting our standard definitions of $\alpha = R/2L$ $\omega_0 = 1/\sqrt{LC}$ and $\omega_d = \sqrt{(\omega_0^2 - \alpha^2)}$

The roots a and b are given by $a = \alpha + j\omega_d$ and $b = \alpha - j\omega_d$

Then I(s) can be expressed as $I(s) = \frac{(V/L)}{(s+\alpha - j\omega d)(s+\alpha + j\omega d)} = \frac{K3}{(s+\alpha - j\omega d)} + \frac{K3*}{(s+\alpha + j\omega d)}$

Here $K3=(s+\alpha-j\omega d).I(s)\,|_{s=-\alpha+j\omega d} = \frac{(V/L)}{(s+\alpha+j\omega d)}\,|_{s=-\alpha+j\omega d} = \frac{(V/L)}{2j\omega d}$

Therefore: $K3 = \frac{(V/L)}{2j\omega d}$ and $K3* = -\frac{(V/L)}{2j\omega d}$

Now substituting these values K3 and K3* in the above expanded equation for I(s) we get

$$I(s) = \frac{(V/L)}{2j\omega d}\frac{1}{(s+\alpha-j\omega d)} - \frac{(V/L)}{2j\omega d}\frac{1}{(s+\alpha+j\omega d)}$$

And now taking inverse transform of I(s) we get

$$i(t) = \frac{(V/L)}{2j\omega d} e^{-\alpha t}.e^{j\omega d t} - \frac{(V/L)}{2j\omega d} e^{-\alpha t}.\ e^{-j\omega d t}$$

$$i(t) = \frac{(V/L)}{\omega d} e^{-\alpha t} [\ (e^{j\omega d t} - e^{-j\omega d t})/2j]$$

$$i(t) = \frac{(V/L)}{\omega d} e^{-\alpha t} \sin\omega_d t$$

Summary of important formulae and equations:

RL circuit with external DC excitation (Charging Transient) :

- $i(t) = V/R\ [1 - e^{-t./\tau}]$
- $v_L(t) = V\ (e^{-t./\tau})$
- $v_R(t) = i(t).R = V\ [1 - e^{-t./\tau}]$

Source free RL circuit (Decay Transients) :

- $i(t) = (V/R)\ .\ e^{-t/\tau}$; $v_R(t) = R.i(t) = Ve^{-t/\tau}$ and $v_L(t) = -Ve^{-t/\tau}$

RC circuit with external DC excitation (Discharge Transients):

- $v_C(t) = V(1 - e^{-t/RC})$
- $v_R(t) = V.\ e^{-t/RC}$
- $i(t) = (V/R)\ e^{-t/RC}$

Source free RC circuit (Discharge transients) :

- $v_C(t) = Ve^{-t/\tau}$; $v_R(t) = -Ve^{-t/\tau}$ and $i(t) = v_R(t)/R = (-V/R)e^{-t/\tau}$

Series RLC circuit: For this circuit three solutions are possible:

1. $\alpha > \omega 0$, i.e when $(R/2L)^2 > 1/LC$, s_1 and s_2 will both be negative real numbers, leading to what is referred to as an *over damped response* given by :

$$i\ (t) = A_1e^{s1t} + A_2e^{s2t}$$

2. $\alpha = \omega_0$, , i.e when $(R/2L)^2 = 1/LC$ s_1 and s_2 are equal which leads to what is called a *critically damped response* given by :

$$i\ (t) = e^{-\alpha t}(A_1t + A_2)$$

3. $\alpha < \omega 0$, i.e when $(R/2L)^2 < 1/LC$ both s_1 and s_2 will have nonzero imaginary components, leading to what is known as an *under damped response* given by :

$$i(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

where :

- $\alpha = (R/2L)$ and is called the exponential damping coefficient
- $\omega 0 = 1/\sqrt{LC}$ and is called the resonant frequency
- $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ and is called the *natural resonant frequency*

Illustrative Examples:

Example 1:Find the current in a series RL circuit having R =2Ω and L = 10H when a DC voltage V of 100V is applied. Find the value of the current 5 secs. after the application of the DC voltage.

Solution: This is a straightforward problem which can be solved by applying the formula. First let us find out the *Time constant* τ of the series LR circuit which is given by $\tau = L/R$ secs.

∴ $\tau = 10/2 = 5$ secs

The current in a series LR circuit after the sudden application of a DC voltage is given by :

$$i(t) = V/R\,(1 - e^{-t/\tau})$$

∴ $i(t)$at 5 secs $= 100/2\,(1 - e^{-5/5}) = 5\,(1 - e^{-1}) = 50\,(1 - 1/e) = 31.48$

∴ **i(t)at 5 secs = 31.48 Amps**

Example2: A series RL circuit has R=25Ω and L=5Henry. A dc voltage V of 100V is applied to this circuit at t = 0 secs. Find :

(a) The equations for the charging current , and voltage across R & L
(b) The current in the circuit 0.5 secs after the voltage is applied.
(c) The time at which the drops across R and L are equal.

Solution: The solutions for (a) and (b) are straightforward as in the earlier problem. (a)*Time constant* τ of the series LR circuit which is given by $\tau = L/R$ secs

∴ $\tau = 5/25 = 1/5$ secs

- The charging current is given by $i(t) = V/R(1 - e^{-t/\tau})$

  It is also given by $i(t) = I(1 - e^{-t/\tau})$ where I is the final steady state current and is equal to V/R

  $$= 100/25\,(1 - e^{-t/(1/5)}) = 4\,(1 - e^{-5t}) \text{ Amps}$$

  $$i(t) = 4\,(1 - e^{-5t}) \text{ Amps}$$

- The voltage across R is given by $v_R = i(t).R = V/R\,(1 - e^{-t/\tau}).R = V\,(1 - e^{-t/\tau})$

  $$v_R = 100\,(1 - e^{-5t})$$

- The voltage drop across L can be found in two ways.
  1. Voltage across Inductor $v_L = L\,di/dt$

2. Voltage across Inductor $v_L = V - v_R$
   But it is easier to find using the second method. $\therefore v_L = 100 - 100\,(1 - e^{-5t})$
   $v_L = 100.\,e^{-5t}$

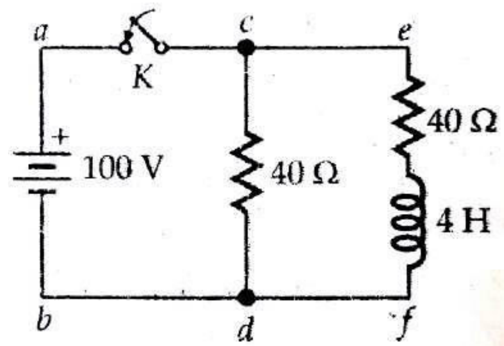
(b) At time t= 0.5 secsi(t) = $4\,(1 - e^{5t}) = 4\,(1 - e^{-2.5}) = 3.67$ Amps

(c) To find out the time at which the voltages across the Inductor and the Resistor are equal we can equate the expressions for $v_R = 100\,(1 - e^{-5t})$ and $v_L = 100.e^{-5t}$ and solve for t. But the simpler method is, we know that since the applied voltage is 100 V the condition $v_R = v_L$ will also be satisfiedwhen $v_R = v_L = 50$ V. i.e $v_R = 100(1 - e^{-5t}) = 50$ volts and $v_L = 100.e^{-5t} = 50$ V. We will solve the second equation [$v_L = 100.\,e^{-5t} = 50$ V] to get t which is easier.

$e^{-5t} = 50/100 = 0.5$.
Taking natural logarithm on both sides we get:
$-5t\,.\ln(e) = \ln 0.5$ i.e $-5t\,.1 = -0.693$ i.e $t = 0.693/5 = 0.139$ secs

∴ *The voltages across the resistance and the Inductance are equal at time t = 0.139 secs*

Example 3: In the figure shown below after the steady state condition is reached , at time t=0 the switch K is suddenly opened. Find the value of the current through the inductor at time t = 0.5 seconds.



Solution: The current in the path *acdb* (through the resistance of 40 Ω alone) is 100/40 = 2.5Amps.(Both steady state and transient are same)
The steady state current through the path *aefb* (through the resistance of 40 Ω and inductance of 4H) is also = 100/40 = 2.5 Amps.

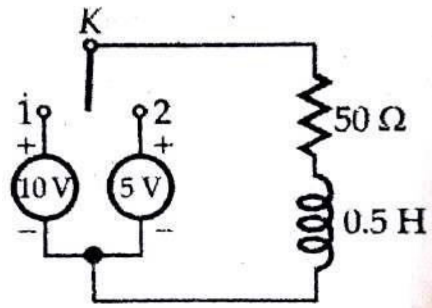
Now when the switch K is suddenly opened, the current through the path *acdb*(through the resistance of 40 Ω alone) immediately becomes zero because this path contains only resistance. But the current through the inductor decays gradually but now through the different path *efdce* The decay current through a closed RL circuit is given by $I.e^{-t/\tau}$ where I is the earlier steady state current of 2.5 amps through L and $\tau = L/R$ of the decay circuit. *It is to be noted carefully here that in the decay path both resistors are there and hence R = 40+40 = 80Ω*
Hence $\tau = L/R = 4/80 = 0.05$ secs

Hence the current through the inductor at time 0.5 secs is given by i(t) $_{@0.5secs}$ =2.5.e $^{-0.5/0.05}$
i.e i(t) $_{@0.5secs}$ = 2.5.e $^{-10}$

i.e i(t) $_{@0.5secs}$= 1.14x10 $^{-4}$ Amps

Example 4: In the circuit shown below the switch is closed to position 1 at time t = 0 secs. Then at time t = 0.5 secs the switch is moved to position 2. Find the expressions for the current through the circuit from 0 to 0. 5 msecs and beyond 0. 5 msecs.

Solution: The time constant τ of the circuit in both the conditions is same and is given by τ = L/R = 0.5/50 = 0.01 secs



1. During the time t=0 to 0.5 msecs. i(t) is given by the standard expression for growing current through a L R circuit: i(t)$_{during\ 0\ to\ 0.5\ msecs}$ = V/R (1—e $^{-t/\tau}$)

i(t)$_{during\ 0\ to\ 0.5\ msecs}$ = V/R (1—e $^{-t/0.01}$) Amps

And the current i(t) $_{@\ t=0.5\ msecs}$ = 10/50 (1-- e $^{-0.5x10-3/0.01}$) = 0.2 (1 – e$^{-0.05}$) = 9.75 mA

i(t) $_{@t=0.5msecs}$ *= 9.75 mA and this would be the initial current when the switch is moved to position 2*

2. During the time beyond 0.5 msecs (switch is in position 2): The initial current is 9.75 mA . The standard expression for the growing current *i(t) = V/R (1—e* $^{-t/\tau}$ *)* is not applicable now since it has been derived with initial condition of i(t) =0 at t=0 where as the initial condition for the current i(t) now in position 2 is 9.75 mA . Now an expression for i(t) in position 2 is to be derived from first principles taking fresh t=0 and initial current i(0) as 9.75mA.
The governing equation in position 2 is given by :

$$50i+0.5di/dt = 5$$

We will use the same *separation of variables method* to solve this differential equation. Dividing the above equation by 0.5, then multiplying by dt and separating the terms containing the two variables i and t we get:
100i + di/dt = 10 i.e 100i.dt +di = 10.dt i.e di = dt (10 – 100i) i.e di/ (10 – 100i) = dt Now integrating on both sides we get

--1/100 ln (10 -- 100i) = t + K ----------------- (1)

The constant K is now to be evaluated by invoking the new initial condition i(t) = 9.75 mAat t =0

$--1/100 \ln(10 - 100 \times 9.75 \times 10^{-3}) = K = --1/100 \ln(10 - 0.975) = --1/100 \ln(9.025)$

Substituting this value of K in the above equation (1) we get

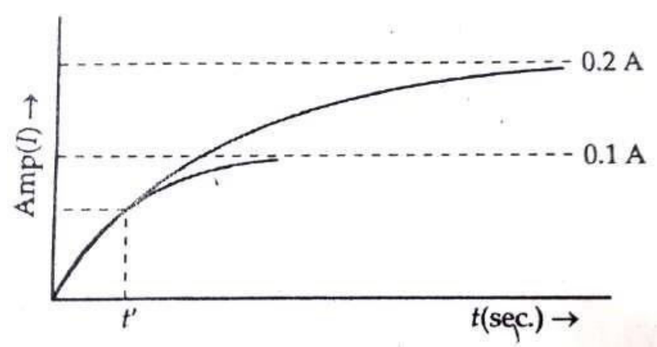
$$--1/100 \ln(10 -- 100i) = t --1/100 \ln(9.025)$$
$$--1/100 \ln(10 -- 100i) + 1/100 \ln(9.025) = t$$
$$--1/100 [\ln(10 -- 100i) -- \ln(9.025)] = t$$
$$--1/100 . \ln[(10 -- 100i) / (9.025)] = t$$
$$\ln[(10 -- 100i)/(9.025)] = --100t$$

Taking antilogarithm to base e on both sides we get:

$$(10 -- 100i)/(9.025)] = e^{--100t} ($$
$$10 -- 100i) = 9.025 \times e^{--100t} ($$
$$10 --9.025 \times e^{--100t}) = 100i$$
$$i = (10 --9.025 \times e^{--100t})/ 100 = 10/100 -- 9.025 \times e^{--100t} /100$$

And finally $i = 0.1 - 0.09. e^{--100t}$

The currents during the periods t = o to 0.5 mses and beyond t = 0.5msec are shown in the figure below. Had the switch been in position 1 all through, the current would have reached the steady state value of 0.2 amps corresponding to source voltage of 10 volts as shown in the top curve. But since the switchis changed to position 2 the current changed it's path towards the new steady state current of 0.1 Amps corresponding the new source voltage of 5 Volts from 0.5 msecs onwards.

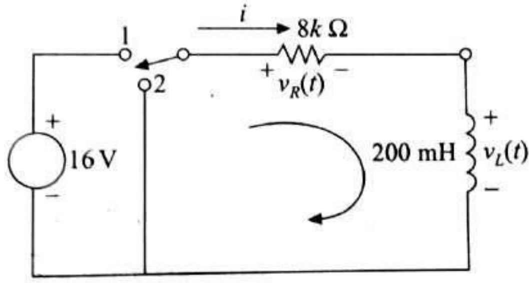


Example 5: In the circuit shown below the switch is kept in position 1 upto 250 μsecs and then moved to position 2. Find

(a) The current and voltage across the resistor at t = 100 μsecs

(b) The current and voltage across the resistor at t = 350 μsecs

Solution : The time constant τ of the circuit is given by τ = L/R = 200mH/8KΩ = 25 μsec and is same in both the switch positions.



(a) The current in the circuit upto 250 μsec (till switch is in position 1) is given by : $i(t)_{growing} = V/R\,(1 - e^{-t/\tau}) = (16/8)X10^{-3}\,(1 - e^{-t/25\,x10^{--6}}) = 2x(1 - e^{-t/25\,x10^{-6}})$ mA

- The current in the circuit @100μsec is given by
  $i(t)_{@100\ \mu sec} = 2x\ (1 - e^{-100\ \mu sec\ /\ 25\ \mu sec})$ mA = $2x(1 - e^{-4})$ mA = 1.9633 mA

$$i(t)_{@100\ \mu sec} = 1.9633\ m$$

- The Voltage across the resistoris given by $v_{R@100\ \mu sec} = R\ x\ i(t)_{@100\ \mu sec}$
  $v_{R@100\ \mu sec}$ = 8 KΩ x1.9633 mA = 15.707 V

$$v_{R@100\ \mu sec} = 15.707\ V$$

(b)

- The current in the circuit @350 μsec is the decaying current and is given by:

$i(t)_{Decaying} = I(0).e^{-t/\tau}$ where I(0) is the initial current and in this case it is the growing current @250μsec. (Since the switch is changed @250μsec) The time t is to be reckoned from this time of 250 μsec. Hence t = (350—250) = 100μsec. So we have to calculate first $i(t)_{growing(@250\ \mu sec)}$ which is given by:
$i(t)_{growing(@250\ \mu sec)} = V/R\ (1 - e^{-t/\tau}) = (16/8)X10^{-3}\ (1 - e^{-t/25\ \mu sec}) = 2x(1 - e^{-250/25\ \mu sec})$ mA
$=2x(1 - e^{-10})$ mA = 1.999 mA

$$i(t)_{growing(@250\ \mu sec)} = 1.999\ mA = I(0)$$

Hence $i(t)_{@350\ \mu sec} = I(0).e^{-t/\tau} = 1.99x\ e^{-100\ \mu sec\ /25\ \mu sec}$mA = $1.99x\ e^{-4}$mA = 0.03663 mA

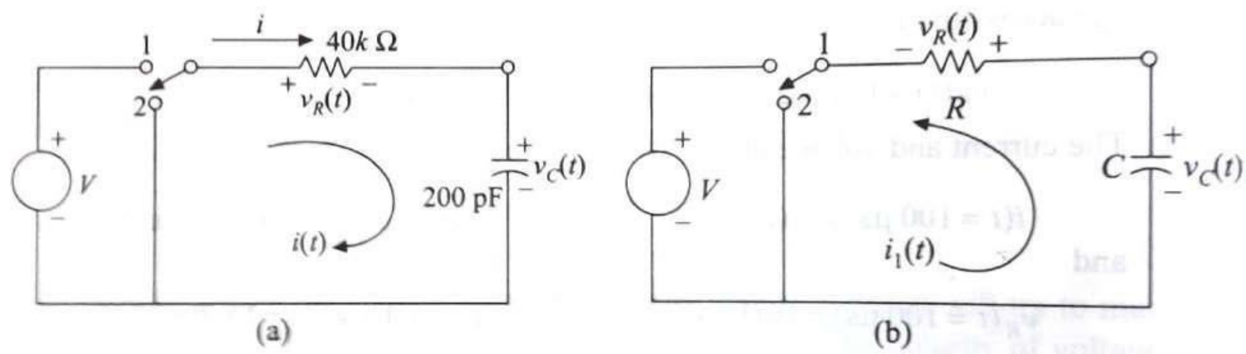
$$i(t)_{@350\ \mu sec} = 0.03663\ mA$$

- The voltage across the resistor v_R @350 μsec = Rxi(t@350 μsec) = 8KΩx0.03663 mA

$$v_{R\ @350\ \mu sec} = 0.293V$$

Example 6: In the circuit shown below the switch is kept in position 1 up to 100 μ secs and then it is moved to position 2 . Supply voltage is 5V DC . Find

a) The current and voltage across the capacitor at t = 40 μ secs
b) The current and voltage across the resistor at t = 150 μ secs



Solution:The time constant τ of the circuit is same in both conditions and is given by $\tau = RC = 40x10^3x200x10x^{-12} = 8$ μsec

a) The time t = 40 μsec corresponds to the switch in position 1 and in that condition the current i(t) is given by the standard expression for charging current

$$i(t) = (V/R)\ [e^{-t/\tau}]$$

$$i(t)_{@40\mu sec} = 5v/40K\Omega\ [e^{-40/8}]\ Amps = 0.125x[e^{-5}]\ mA = 0.84224\ \mu A$$

$$i(t)_{@40\mu sec} = 0.84224\ \mu A$$

The voltage across the capacitor during the charging period is given by $V [1- e^{-t/\tau}]$. $v_C(t)_{@40\mu sec} = 5[1 -- e^{-40/8}] = 5[1 -- e^{-5}] = 4.9663$ Volts

$$v_C(t)_{@40\mu sec} = 4.9663\ Volts$$

b) The time t = 150 μsec corresponds to the switch in position 2 and the current i(t) is given by the discharge voltage expression $i(t) = [v_C(t)_0/R].\ e^{-t/\tau}$

Where $v_C(t)_0$ is the initial capacitor voltage when the switch was changed to position 2 and it is the voltage that has built up by 100 µsec during the charging time (switch in position 1) and hence is given by

$$v_C(t)_{@100\mu sec} = 5[1- e^{-100/8}] \text{ volts} = 5x[1- e^{-12.5}] \text{ Volts} = 4.999 \text{ Volts}$$

And now t=150 µsec from beginning is equal to t = (150-100) = 50 µsec from the time switch is changed to position 2.

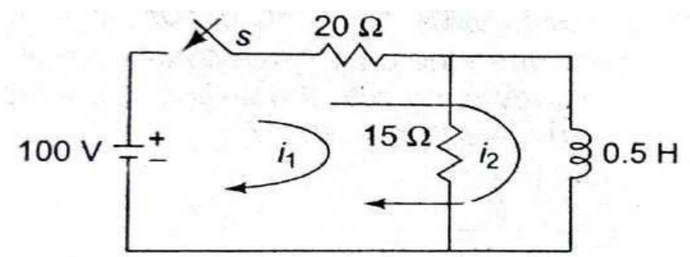
Therefore the current through the resistor at 150 µsec from the beginning = $i(t)_{150\mu sec}$= (4.999/40KΩ). $e^{-t/\tau}$

$i(t)_{150\mu sec} = 0.1249 \times e^{-50/8} = 0.241$ µA

$$i(t)_{150\mu sec} = 0.241 \ \mu A$$

And the voltage across the resistor = R x i(t) = 40KΩ x 0.241 µA = 0.00964v

Example 7: In the circuit shown below find out the expressions for the current i_1 and i_2 when the switch is closed at time t= 0



Solution: It is to be noted that in this circuit there are two current loops 1 and 2 . Current i_1 alone flows through the resistor 15 Ω and the current i_2 alone flows through the inductance0.5 H where as both currents i_1 and i_2 flow through the resistor 20 Ω. Applying KVL to the two loops taking care of this point we get

$20(i_1 + i_2) + 15 i_1 = 100$ i.e $35 i_1 + 20 i_2 = 100$ -------- (1)

and $20(i_1 + i_2) + 0.5\, di_2/dt = 100$; $20 i_1 + 20 i_2 + 0.5\, di_2/dt = 100$------------- (2)

Substituting the value of i_1 = [100/35 – (20/35) i_2] = 2.86 – 0.57 i_2 obtained from the above equation (1) into equation (2) we get :

$20 [2.86 – 0.57 i_2] + 20 i_2 + 0.5 (di_2/dt) = 100$

$57.14 – 11.4 i_2 + 20 i_2 + 0.5 (di_2/dt) = 100$

$(di_2/dt) i_2 + 17.14 i_2 = 85.72$

The solution for this equation is given by $i_2(t) = K.\ e^{-17.14t} + 85.72/17.14$ and the constant K can be evaluated by invoking the initial condition. The initial current through the inductor = 0 at time t = 0 .

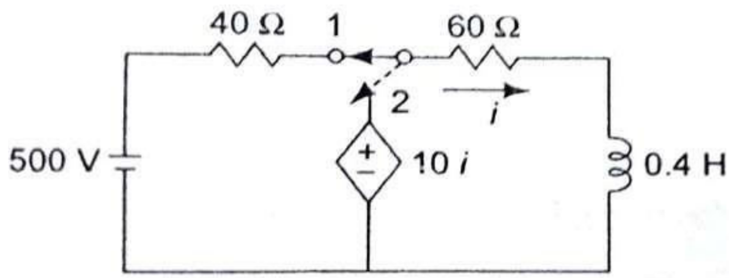
Hence K = -- 85.72/17.14 = -- 5

Therefore $i_2(t) = 5\ (\ 1-- e^{-17.14t}\ )$ Amps

And current $i_1(t) = 2.86 - 0.57\ i_2 = 2.86 - 0.57\ [5\ (\ 1-- e^{-17.14t}\ )\ ] = 0.01 + 2.85\ e^{-17.14t}$ Amps

And current $i_1(t) = 0.01 + 2.85\ e^{-17.14t}$ Amps

Example 8 : In the circuit shown below find an expression for the current i(t) when the switch is changed from position 1 to 2 at time t= 0 .



Solution: The following points are to be noted with reference to this circuit:

- When the switch is changed to position 2 the circuit is equivalent to a normal source free circuit but with a current dependent voltage source given as 10i.
- The initial current in position 2 is same as the current when the switch was in position 1 (for a long time) and is given by $I_0 = 500/(40+60) = 5$ Amps

The loop equation in position 2 is given by : 60i + 0.4 di/dt = 10i i.e (50/0.4)i + di/dt = 0

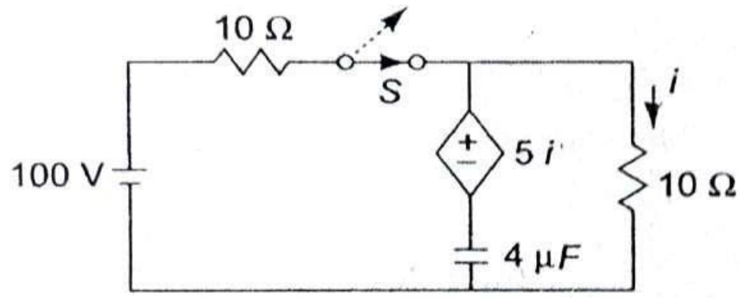
Writing the equation in the 's' notation where 's' is the operator equivalent to (d/dt) we get (s+ 125

) i = 0 and the characteristic equation will be (s+ 125) = 0

Hence the solution i(t) is given by $i(t) = K.\ e^{--125t}$. The constant K can be evaluated by invoking the initial condition that $i(t)_{@ t=0}$ is equal to $I_0 = 5$ amps .Then the above equation becomes:

$5 = K\ .\ e^{—125X0}$ i.e K = 5 and hence the current in the circuit when the switch is changed to position 2 becomes: $i(t) = 5.\ e^{--125t}$ Amps

Example 9 : In the circuit shown below find an expression for the current i(t) when the switch is opened at time t= 0



Solution: The following points may be noted with reference to this circuit:

- When the switch is opened the circuit is equivalent to a normal source free circuit but with a current dependent voltage source given as 5i.
- The initial current I_0 when the switch is opened is same as the current when the switch was closed for a long time and is given by $I_0 = 100/(10+10) = 5$ Amps

The loop equation when the switch is opened is given by :

$$(1/4x10^{-6})\int idt + 10i = 5i$$
$$(1/4x10^{-6})\int idt + 5i = 0$$

Differentiating the above equation we get :

$$5.(di/dt) + (1/4x10^{-6})i = 0 \qquad \text{i.e.} = (di/dt) + (1/20 \text{ x } 10^{-6})i = 0$$

Writing the above equation in the 's' notation where 's' is the operator equivalent to (d/dt) we get

$(s+ 1/20 \text{ x } 10^{-6}) i = 0$ and the characteristic equation will be $(s+ 1/20 \text{ x } 10^{-6}) = 0$

The solution i(t) is given by $i(t) = K . e^{-t/20x10^{-6}}$. The constant K can be evaluated by invoking the initial condition that $i(t)_{@ t=0}$ is equal to $I_0 = 5$ amps . Then the above equation becomes:

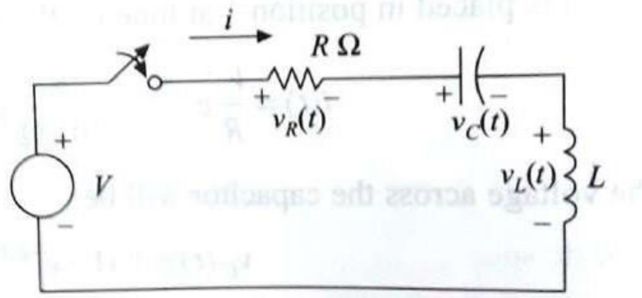
$5 = K . e^{-t/20x10^{-6}}$ i.e K = 5 and hence the current in the circuit when the switch is opened becomes:

$$i(t) = 5. e^{-t/20 \text{ x } 10^{-6}} \text{ Amps}$$

Example 10: A series RLC circuit as shown in the figure below has R = 5Ω,L= 2H and C = 0.5F.The supply voltage is 10 V DC . Find

a) The current in the circuit when there is no initial charge on the capacitor.
b) The current in the circuit when the capacitor has initial voltage of 5V
c) Repeat question (a) when the resistance is changed to 4 Ω

d) Repeat question (a) when the resistance is changed to 1 Ω



Solution: The basic governing equation of this series circuit is given by :

$$Ri + 1/C\int idt + L.\ (di/dt) = V$$

On differentiation we get the same equation in the standard differential equation form

$$L(d^2i/dt^2) + R(di/dt) + (1/C)i = 0$$

By dividing the equation by L and using the operator 's' for d/dt we get theequation in the form of characteristic equation as :

$$[s^2 + (R/L)s + (1/LC)] = 0$$

Whose roots are given by:

$$s_1, s_2 = -R/2L \pm \sqrt{[(R/2L)^2 - (1/LC)]} = -\alpha \pm \sqrt{(\alpha^2 - \omega_0^2)}$$

and three types of solutions are possible.

1. $\alpha > \omega 0$, i.e when $LC > (2L/R)^2$ s_1 and s_2 will both be negative real numbers, leading to what is referred to as an *over damped response*given by :

$$i\ (t) = A_1e^{s1t} + A_2e^{s2t}$$

2. $\alpha = \omega_0$, , i.e when $LC = (2L/R)^2$ s_1 and s_2 are equal which leads to what is called a *critically damped response*given by :

$$i\ (t) = e^{-\alpha t}(A_1t + A_2)$$

3. $\alpha < \omega 0$, i.e when $LC < (2L/R)^2$ both s_1 and s_2 will have nonzero imaginary components, leading to what is known as an *under damped response*given by :

$$i\ (t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

where$\omega_d$is called *natural resonant frequency* and is given given by:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

The procedure to evaluate the complete solution consists of the following steps for each part of the question:

1. We have to first calculate the roots for each part of the question and depending on to which case the roots belong we have to take the appropriate solution .
2. Then by invoking the first initial condition i.e i = 0 at t=0 obtain the first relation between A_1 and A_2or one of its values.
3. If one constant value is obtained directly substitute it into the above solution and take its first derivative. Or else directly take the first derivative of the above solution

4. Now obtain the value di/dt $_{@\ t=0}$ from the basic RLC circuit equation by invoking the initial conditions of $v_{C@\ t=0}$ and i(t) $_{@\ t=0}$. Now equate this to the differential of the solution for i(t) to get the second relation between A_1 and A_2(or the second constant . Now using these two equations we can solve for A_1 and A_2 and subsititute in the solution for i(t) to get the final solution.

(a) $s_1,s_2 = -R/2L \pm \sqrt{(R/2L)^2 - (1/LC)} = (-5/2x2) \pm \sqrt{[(5/2x2)^2 - (1/2x0.5)]} = -1.25 \pm 0.75$.
i.e. $s_1 = --0.5$ and $s_2 = --2$

In this case the roots are negative real numbers and the solution is given by: $i(t) = A_1e^{s1t} + A_2e^{s2t} = A_1e^{-0.5t} + A_2e^{-2t}$ ----------------- (1)
Now we will apply the first initial condition i.e i(t) = 0 at t=0 .Then we get
$0 = A_1e^{-0.5x0} + A_2e^{-2x0}$ i.e. $A_1 + A_2 = 0$

The basic equation for voltage in the series RLC circuit is given as : V =

R.i(t) + v_C(t) + L.(di/dt) i.e di/dt = 1/L [V -R.i(t) - v_C(t)
At time t=0 we get

$(di/dt)_{@\ t=0}$ = 1/L [V -R.i(t=0) - v_C(t=0)]-------------- (2)

But we know that the voltage across the capacitor and current are zero at time t=0 .

Therefore $(di/dt)_{@\ t=0}$ = V/L = 10/2 = 5--------------------- (3)

Now the equation for i(t) at equation (1) is differentiated to get (di/dt) =
$-0.5A_1e^{-0.5t} - 2A_2e^{-2t}$
and the above value of $(di/dt)_{@\ t=0} = 5$ is substituted in that to get the second equation with A_1 and A_2
$(di/dt)_{@\ t=0} = 5 = -0.5A_1e^{-0.5x0} - 2A_2e^{-2x0} = -0.5A_1 - 2A_2$
Now we can solve the two equations for A_1 and A_2

$A_1 + A_2 = 0$ and $-0.5A_1 - -2A_2 = 5$ to get $A_1 = 10/3$ and $A_2 = ---\ 10/3$

And the final solution for i(t) is : $(10/3)[e^{-0.5t} - e^{-2t}]$ Amps

(b) At time t=0 the voltage across the capacitor = 5V ie. $v_{C(t=0)}$ = 5V. But i(t=0) is still = 0.using these values in the equation (2) above we get
$(di/dt)_{@\ t=0}$ = ½ (10-5) = 2-5
Then the two equations in A_1 and A_2 are $A_1 + A_2 = 0$ and $-0.5A_1 - -2A_2 = 2.5$
Solving these two equations we get $A_1 = 5/3$ and $A_2 = -5/3$

And the final solution for i(t) is : $(5/3)[e^{-0.5t} - e^{-2t}]$ Amps

(c) The roots of the characteristic equation when the Resistance is changed to 4 $s_1,s_2 = -R/2L \pm \sqrt{(R/2L)^2 - (1/LC)} = (-4/2x2) \pm \sqrt{[(4/2x2)^2 - (1/2x0.5)]} = -1.0$
i.e the roots are real and equal and the solution is given by

$i(t) = e^{-\alpha t}(A_1t + A_2) = e^{-1t}(A_1t + A_2)$------------(4)

Now using the initial condition i(t) = 0 at time t=0 we get $A_2 = 0$

We have already found in equation (3) for the basic series RLC circuit $(di/dt)_{@\ t=0} = 5$

Now we will find di(t)/dt of equation (4) and equate it to the above value. $di/dt = -e^{-1t}(A_1t + A_2) + e^{-1t}(A_1) = e^{-1t}[A_1 - A_1t - A_2]$ and

$(di/dt)_{@t=0} = e^{-1x0}[A_1 - A_1x0 - A_2]$ i.e

$A_1 - A_2 = 5$ Therefore A1 =5 and A2 = 0

And the final solution for i(t) is $i(t) = 5te^{-1t}$ Amps

(d) Roots of the characteristic equation when the resistance is changed to 1 Ω are :

$s_1,s_2 = -R/2L \pm \sqrt{[(R/2L)^2 - (1/LC)]} = (-1/2x2) \pm \sqrt{[(1/4)^2 - (1/2x0.5)]} = -0.25 \pm j0.94$

The roots are complex and so the solution is then given by : $i(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

Where $\alpha = 0.25$ and $\omega_d = 0.9465$

Now we will apply the initial conditions to find out the constants A_1 and A_2

First initial condition is $i(t)_{@t=0} = 0$ applying this into the equation : $i(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$ we get A1 = 0 and using this value of A1 in the abve equation for i(t) we get

$i(t) = e^{-\alpha t}(A_2 \sin\omega_d t)$

We have already obtained the second initial condition as $di(t)/dt_{@t=0} = 5$ from the basic equation of the series RLC circuit. Now let us differentiate above equation for current i.e : $i(t) = e^{-\alpha t}(A_2 \sin\omega_d t)$ and equate it to 5 to get the second constant A_2

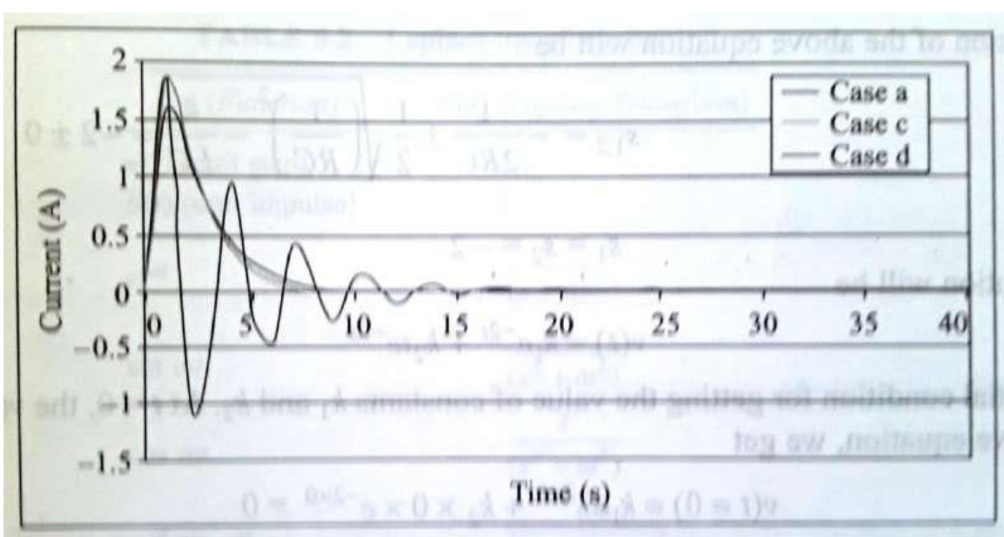
$di(t)/dt = e^{-\alpha t}(A_2\,\omega_d \cos\omega_d t) + (A_2 \sin\omega_d t)\,.-\alpha.\,e^{-\alpha t}$ $di(t)/dt\ @t=0 = A_2.\ \omega_d = 5$

i.e $A_2 = 5 / \omega_d = 5/0.94 = 5.3$

Now using this value of A_2 and the values of $\alpha = 0.25$ and $\omega_d = 0.94$ in the above expression for the current we finally get :

$$i(t) = e^{-0.25t}(2.569 \sin 1.9465t)$$

The currents in all the three different cases (a), (c) and (d) are shown below :



Conclusion:

The most important facts and results discussed in the chapter can be summarized as follows:

- Transients in electric circuits occur due to the presence of energy storage elements (i.e., inductors and capacitors).
- Transients in electric circuits can be excited by initial conditions, by sources, or by both.

Analysis of transients can be broken down into two major steps:

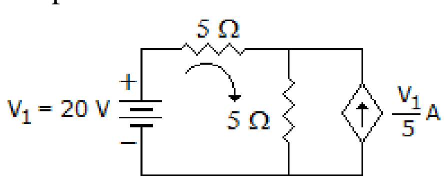
1. Determination of initial conditions for the energy storage elements by using the continuity of voltage across a capacitor and the continuity of current through an inductor.
2. Analysis of electric circuits after switching. This step normally involves the solution of initial value problems for ordinary differential equations.

Reference:

[1].Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi,2000
[2]. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education 2004
[3]. Paranjothi SR, "Electric Circuits Analysis," New Age International Ltd., New Delhi, 1996.

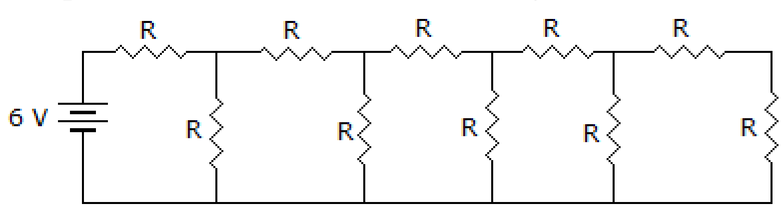
Post Test MCQs:

1. The dependent current source shown in given figure.



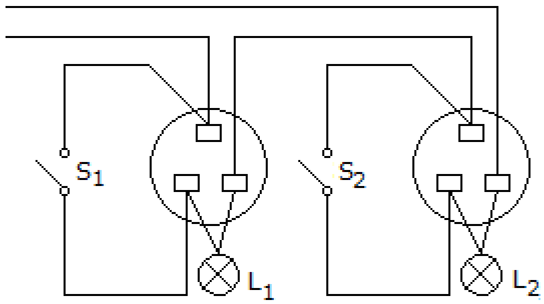
a. delivers 80W
b. absorbs 80W
c. deliver 40W
d. absorbs 40W

2. The input resistance of the network in figure is



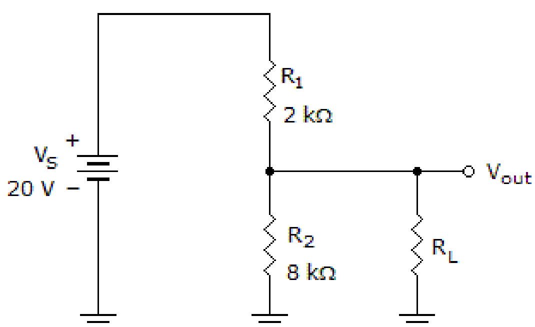
a. 10R
b. $\frac{13}{8}$ R
c. $\frac{34}{21}$ R
d. $\frac{89}{55}$ R

3. The closing of switch S_1 and S_2 in figure will light up



a. lamp L_1 only
b. lamps L_1 and L_2
c. lamp L_2 only
d. none of them

4.



If the load in the given circuit is 120 kΩ, what is the loaded output voltage?

a. 4.21 V
b. 15.79 V
c. 16 V
d. 19.67 V

5 The current flowing through an unloaded voltage divider is called the:

a. resistor current
b. load current
c. bleeder current
d. voltage current

UNIT – V
NETWORK SYNTHESIS

AIM :

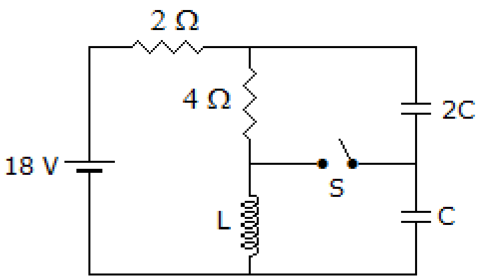
To understand the significance of network synthesis

Pre-Requisites:

Knowledge of Basic Mathematics – II & Basic Electronics Engineering

Pre - MCQs*:*

1, In figure, the switch S is initially open and steady state conditions are reached. At *t* = 0 switch is closed. The initial current through 2C capacitor is

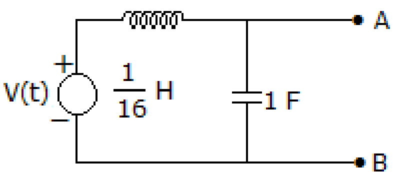


a. zero
b. 1A
c. 2A
d. 3A

2. For an RC driving point impedance function, the poles and zeros

a. should alternate on real axis
b. should alternate only on negative real axis
c. should alternate on imaginary axis
d. none of the above

3. The circuit in figure will act as ideal current source with respect to terminals A and B when frequency is



a. 0
b. 1 rad/sec
c. 4 rad/sec
d. 16 rad/sec

Introduction:

Network synthesis is a design technique for linear electrical circuits. **Synthesis** starts from a prescribed impedance function of frequency or frequency response and then determines the possible **networks** that will produce the required response.

HurwitzPolynomial

A polynomial *p(s)* is said to be *Hurwitz* if all the roots of *p(s)* are located in the open left half (LH) s-plane (not including the imaginary axis).Let p(s) be the polynomial in question. Assume first that p(s) is neither an even nor an odd polynomial. To test whether such a polynomial p(s) is indeed a Hurwitz polynomial, we may use *the Hurwitz test*.

- First decompose p(s) into its even and odd parts, M(s) and N(s), respectively, as p(s) = M(s) + N(s).
- Using M(s) and N(s) we form the test ratio T(s), whose numerator has a higher degree than that of its denominator. Suppose that p(s) is a polynomial of degree d. Then

$$T(s) = \frac{N(s)}{M(s)} \quad \text{if } d \text{ is an odd integer} \qquad \text{(4-8a)}$$

$$T(s) = \frac{M(s)}{N(s)} \quad \text{if } d \text{ is an even integer} \qquad \text{(4-8b)}$$

Next, we perform the continued fraction expansion about infinity on the test ratio T(s), removing one pole at a time in the form of a quotient qs, resulting in:

$$T(s) = q_1 s + \cfrac{1}{q_2 s + \cfrac{1}{q_3 s + \cfrac{1}{\ddots + \cfrac{1}{q_{\hat{d}} s}}}} \qquad \text{(4-9)}$$

where $q_i s$ is the ith quotient, and q_i, is the associated coefficient.

If there is one or more quotients with negative coefficients, then p(s) is neither a Hurwitz nor a modified Hurwitz polynomial. On the other hand, if there are d quotients ($d = \hat{d}$) and every quotient has a positive coefficient, then p(s) is a Hurwitz polynomial.

Finally, if the number of quotient $\hat{d}$ is less than d but every quotient has a positive coefficient, this means that there is a common factor k(s) between M(s) and N(s). Hence, we can write p(s) as:

$$p(s) = k(s)\,[\hat{M}(s) + \hat{N}(s)] = k(s)\hat{p}(s) \qquad \textbf{(4-10)}$$

where $M(s) = k(s)\hat{M}(s)$, $N(s) = k(s)\hat{N}(s)$, **and** $\hat{p}(s) = \hat{M}(s) + \hat{N}(s)$.

Because all the $\hat{d}$ quotients of T(s) have positive coefficients, the polynomial p(s) in (4-10) is Hurwitz. Thus, if k(s) is a modified Hurwitz polynomial [i.e., if all the roots of k(s) are simple and purely imaginary], then p(s) is a modified Hurwitz polynomial.

A procedure to determine if k(s) is a modified Hurwitz polynomial is described in the following in conjunction with the case when p(s) is either an even or an odd polynomial.

Suppose now that p(s) is either an even or an odd polynomial of degree d. is a modified Hurwitz polynomial if and only if p(s) has only simple and imaginary axis roots (including the origin).

To determine if p(s) is a modified Hurwitz polynomial, we form a test ratio $\hat{T}(s)$:

$$\hat{T}(s) = \frac{p(s)}{(d/ds)\,p(s)} = \frac{p(s)}{p'(s)} \qquad \textbf{(4-12)}$$

and perform the continued fraction expansion about infinity on $\hat{T}(s)$, as in (4-9). Then p(s) is a modified Hurwitz polynomial if and only if there are d quotients in the expansion and each quotient has a positive coefficient.

In the case when p(s) is either an even or an odd polynomial, if there is one or more negative coefficient in the continued fraction expansion of T^(s), then p(s) has a RH s-plane root; and if all coefficients are positive but there are only d^ < d quotients, then all roots of p(s) are on the imaginary axis of the s-plane, but p(s) has non-simple or multiple roots. Either situation implies that p(s) is not a modified Hurwitz polynomial.

Example **Determine if**

$$p(s) = s^4 + 3s^3 + 5s^2 + 5s + 2 \qquad \textbf{(4-13)}$$

is a Hurwitz polynomial.

Because d -— 4 is even, the test ratio is

$$T(s) \triangleq \frac{M\{s)}{N(s)} - \frac{s^* - 5s^2 -1- 2}{3s^3 + 5s} \quad \text{(V14)}$$

Clearly, at r = T{s) en [i.e., T s) has a pole at infinity]. Extracting this pole at infinity in the form of a quotient, we obtain

$$T(s) = {}^1 s + \frac{1}{T_1(s)} \quad \text{(4-15)}$$

where (1J3)z is the first quotient, 1/3 is its coefficient, and

$$\frac{1}{s)} \quad T\{s) - \frac{i}{3}s - \frac{(10/3)s^2 -£ 2}{3W+56}$$

is the remainder. Hence,

$$T_1(s) \quad \frac{3s^\circ + 5s}{(10/3)s^\circ -1- 2} \quad \text{(4-16)}$$

Observe that *Th{‹x›)* m. Thus, we can extract a Pole from *Th[s)* in the form of a quotient as we did to F(s). The result is to write Z'i(«)as

$$T\ \{s) - \frac{9}{10}s + \frac{1}{T\ 2\ S)} \quad \text{(4-17)}$$

where (9JI0)x is the second quotient, 9/10 is its coefficient, and 1/F 2(s) is the second remainder. Substituting (4-17) into (4-11),

Routh–Hurwitz stability criterion:

A tabular method can be used to determine the stability when the roots of a higher order characteristic polynomial are difficult to obtain. For an *n*th-degree polynomial

- $D(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$

the table has $n + 1$ rows and the following structure:

a_n	a_{n-2}	a_{n-4}	$\cdots$
a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
b_1	b_2	b_3	$\cdots$
c_1	c_2	c_3	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$

where the elements b_i and c_i can be computed as follows:

$$b_i = \frac{a_{n-1} \times a_{n-2i} - a_n \times a_{n-2i-1}}{a_{n-1}}.$$

$$c_i = \frac{b_1 \times a_{n-2i-1} - a_{n-1} \times b_{i+1}}{b_1}.$$

When completed, the number of sign changes in the first column will be the number of non-negative poles.

In the first column, there are two sign changes, thus there are two non-negative roots where the system is unstable. Sometimes the presence of poles on the imaginary axis creates a situation of marginal stability. The row of polynomial which is just above the row containing the zeroes is called "Auxiliary Polynomial".

- $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0.$

We have the following table:

1	8	20	16
2	12	16	0
2	12	16	0
0	0	0	0

In such a case the Auxiliary polynomial is $A(s) = 2s^4 + 12s^2 + 16.$ which is again equal to zero. The next step is to differentiate the above equation which yields the following polynomial. $B(s) = 8s^3 + 24s^1.$. The coefficients of the row containing zero now become "8" and "24". The process of Routh array is proceeded using these values which yield two points on the imaginary axis. These two points on the imaginary axis are the prime cause of marginal stability.

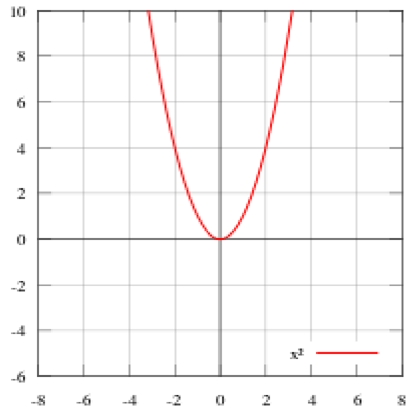
Even and Odd functions :

In mathematics, **even functions** and **odd functions** are functions which satisfy particular symmetry relations, with respect to taking additive inverses. They are important in many areas of mathematical analysis, especially the theory of power series and Fourier series. They are named for the parity of the powers of the power functions which satisfy each condition: the function $f(x) = x^n$ is an even function if n is an even integer, and it is an odd function if n is an odd integer.

Definition and examples

The concept of evenness or oddness is only defined for functions whose domain and range both have an additive inverse. This includes additive groups, all rings, all fields, and all vector spaces. Thus, for example, a real-valued function of a real variable could be even or odd, as could a complex-valued function of a vector variable, and so on. The examples are real-valued functions of a real variable, to illustrate the symmetry of their graphs.

Even functions



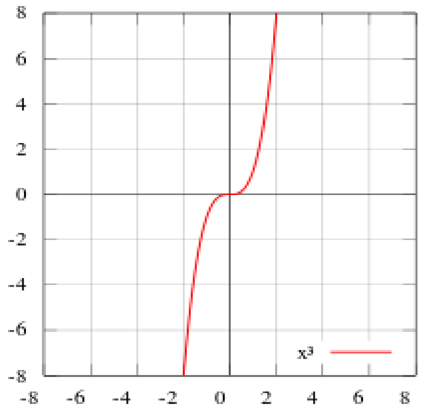
$f(x) = x^2$ is an example of an even function.

Let $f(x)$ be a real-valued function of a real variable. Then f is **even** if the following equation holds for all x and $-x$ in the domain of f:[1]

$$f(x) = f(-x), \qquad f(x) - f(-x) = 0.$$

Geometrically speaking, the graph face of an even function is symmetric with respect to the y-axis, meaning that its graph remains unchanged after reflection about the y-axis. Examples of even functions are $|x|$, x^2, x^4, $\cos(x)$, and $\cosh(x)$.

Odd functions



$f(x) = x^3$ is an example of an odd function.

Again, let $f(x)$ be a real-valued function of a real variable. Then f is **odd** if the following equation holds for all x and $-x$ in the domain of f:[2]

$$-f(x) = f(-x), \qquad f(x) + f(-x) = 0.$$

Geometrically, the graph of an odd function has rotational symmetry with respect to the origin, meaning that its graph remains unchanged after rotation of 180 degrees about the origin. Examples of odd functions are x, x^3, $\sin(x)$, $\sinh(x)$, and $\mathrm{erf}(x)$.

Continuity and differentiability

A function's being odd or even does not imply differentiability, or even continuity. For example, the Dirichlet function is even, but is nowhere continuous. Properties involving Fourier series, Taylor series, derivatives and so on may only be used when they can be assumed to exist.

Generalizations

Irrational functions

The irrational function $Z(s)$ is PR if and only if

- $Z(s)$ is analytic in the open right half s-plane ($\mathrm{Re}[s] > 0$)
- $Z(s)$ is real when s is positive and real
- $\mathrm{Re}[Z(s)] \geq 0$ when $\mathrm{Re}[s] \geq 0$

Matrix-valued functions

A irrational matrix-valued function $Z(s)$ is PR if and only if

- Each element of $Z(s)$ is analytic in the open right half s-plane ($\mathrm{Re}[s] > 0$)
- Each element of $Z(s)$ is real when s is positive and real
- The Hermitian part $(Z(s) + Z^{\dagger}(s))/2$ of $Z(s)$ is positive semi-definite when $\mathrm{Re}[s] \geq 0$

SYNTHESIS OF R-L NETWORK BY FOSTER'S METHOD

Introduction :

By Kirchhoff's voltage law, the voltage across the capacitor, V_C, plus the voltage across the inductor, V_L must equal zero:

$$V_C + V_L = 0.$$

Likewise, by Kirchhoff's current law, the current through the capacitor equals the current through the inductor:

$$i_C = i_L.$$

From the constitutive relations for the circuit elements, we also know that

$$V_L(t) = L\frac{di_L}{dt}$$

and

$$i_C(t) = C\frac{dV_C}{dt}.$$

Differential equation

Rearranging and substituting gives the second order differential equation

$$\frac{d^2i(t)}{dt^2} + \frac{1}{LC}i(t) = 0.$$

The parameter ω_0, the resonant angular frequency, is defined as:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Using this can simplify the differential equation

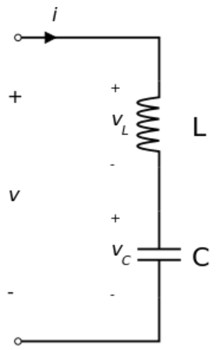
$$\frac{d^2i(t)}{dt^2} + \omega_0^2 i(t) = 0.$$

The associated polynomial is

$$s^2 + \omega_0^2 = 0$$

Thus, $s = +j\omega_0$ $s = -j\omega_0$ where *j* is the imaginary unit.

Series LC circuit



Series LC circuit

In the series configuration of the LC circuit, the inductor *L* and capacitor *C* are connected in series, as shown here. The total voltage *v* across the open terminals is simply the sum of the voltage across the inductor and the voltage across the capacitor. The current *i* into the positive terminal of the circuit is equal to the current through both the capacitor and the inductor.

$$v = v_L + v_C$$

$$i = i_L = i_C$$

Resonance

Inductive reactance magnitude (X_L) increases as frequency increases while capacitive reactance magnitude (X_C) decreases with the increase in frequency. At one particular frequency, these two reactances are equal in magnitude but opposite in sign; that frequency is called the resonant frequency (f_0) for the given circuit.

Hence, at resonance:

$$X_L = -X_C$$

$$\omega L = \frac{1}{\omega C}$$

Solving for ω we have

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

which is defined as the resonant angular frequency of the circuit.

Converting angular frequency (in radians per second) into frequency (in hertz),one has

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

In a series configuration, X_C and X_L cancel each other out. In real, rather than idealised components, the current is opposed, mostly by the resistance of the coil windings. Thus, the current supplied to a series resonant circuit is a maximum at resonance.

- In the limit as $f \to f_0$ current is maximum. Circuit impedance is minimum. In this state, a circuit is called an *acceptor circuit*
- For $f < f_0$, $X_L \ll (-X_C)$. Hence, the circuit is capacitive.
- For $f > f_0$, $X_L \gg (-X_C)$. Hence, the circuit is inductive.

Impedance

In the series configuration, resonance occurs when the complex electrical impedance of the circuit approaches zero.

First consider the impedance of the series LC circuit. The total impedance is given by the sum of the inductive and capacitive impedances:

$$Z = Z_L + Z_C$$

Writing the inductive impedance as $Z_L = j\omega L$ and capacitive impedance as $Z_C = (j\omega C)^{-1}$ and substituting gives

$$Z(\omega) = j\omega L + \frac{1}{j\omega C}.$$

Writing this expression under a common denominator gives

$$Z(\omega) = j\frac{(\omega^2 LC - 1)}{\omega C}.$$

Finally, defining the natural angular frequency as

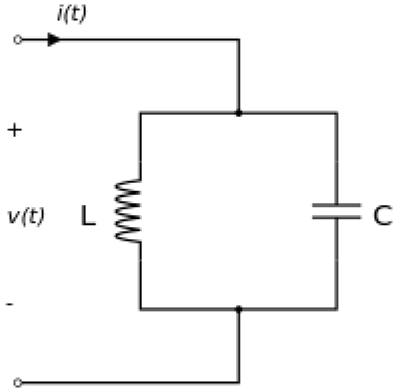
$$\omega_0 = \frac{1}{\sqrt{LC}},$$

the impedance becomes

$$Z(\omega) = jL\left(\frac{\omega^2 - \omega_0^2}{\omega}\right).$$

The numerator implies that in the limit as $\omega \to \pm\omega_0$, the total impedance *Z* will be zero and otherwise non-zero. Therefore the series LC circuit, when connected in series with a load, will act as a band-pass filter having zero impedance at the resonant frequency of the LC circuit.

Parallel LC circuit



Parallel LC Circuit

In the parallel configuration, the inductor *L* and capacitor *C* are connected in parallel, as shown here. The voltage *v* across the open terminals is equal to both the voltage across the inductor and the voltage across the capacitor. The total current *i* flowing into the positive terminal of the circuit is equal to the sum of the current flowing through the inductor and the current flowing through the capacitor.

$$v = v_L = v_C$$

$$i = i_L + i_C$$

Resonance

Let R be the internal resistance of the coil. When X_L equals X_C, the reactive branch currents are equal and opposite. Hence they cancel out each other to give minimum current in the main line. Since total current is minimum, in this state the total impedance is maximum.

Resonant frequency given by: $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$.

Note that any reactive branch current is not minimum at resonance, but each is given separately by dividing source voltage (V) by reactance (Z). Hence I=V/Z, as per Ohm's law.

- At f_0, line current is minimum. Total impedance is maximum. In this state a circuit is called a *rejector circuit*.
- Below f_0, circuit is inductive.
- Above f_0,circuit is capacitive.

Impedance

The same analysis may be applied to the parallel LC circuit. The total impedance is then given by:

$$Z = \frac{Z_L Z_C}{Z_L + Z_C}$$

and after substitution of $Z_L$and $Z_C$and simplification, gives

$$Z(\omega) = -j \frac{\omega L}{\omega^2 LC - 1}$$

which further simplifies to

$$Z(\omega) = -j \left(\frac{1}{C}\right)\left(\frac{\omega}{\omega^2 - \omega_0^2}\right)$$

where

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Note that

$$\lim_{\omega \to \pm\omega_0} Z(\omega) = \infty$$

but for all other values of ω the impedance is finite. The parallel LC circuit connected in series with a load will act as band-stop filter having infinite impedance at the resonant frequency of the LC circuit. The parallel LC circuit connected in parallel with a load will act as band-pass filter.

RC NETWORK SYNTHESIS:

A **resistor–capacitor circuit** (**RC circuit**), or **RC filter** or **RC network**, is an electric circuit composed of resistors and capacitors driven by a voltage or current source. A first order RC circuit is composed of one resistor and one capacitor and is the simplest type of RC circuit.

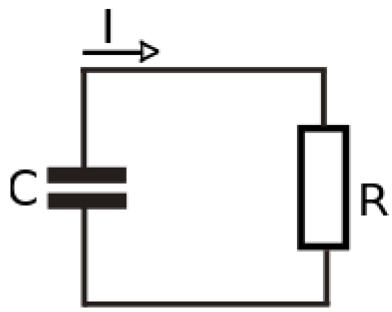
RC circuits can be used to filter a signal by blocking certain frequencies and passing others. The two most common RC filters are the high-pass filters and low-pass filters; band-pass filters and band-stop filters usually require RLC filters, though crude ones can be made with RC filters.

Introduction

There are three basic, linear passive lumped analog circuit components: the resistor (R), the capacitor (C), and the inductor (L). These may be combined in the RC circuit, the RL circuit, the LC circuit, and the RLC circuit, with the abbreviations indicating which components are used. These circuits, among them, exhibit a large number of important types of behaviour that are fundamental to much of analog electronics. In particular, they are able to act as passive filters. This article considers the RC circuit, in both series and parallel forms, as shown in the diagrams below.

This article relies on knowledge of the complex impedance representation of capacitors and on knowledge of the frequency domain representation of signals.

Natural response



RC circuit

The simplest RC circuit is a capacitor and a resistor in series. When a circuit consists of only a charged capacitor and a resistor, the capacitor will discharge its stored energy through the resistor. The voltage across the capacitor, which is time dependent, can be found by using Kirchhoff's current law, where the current charging the capacitor must equal the current through the resistor. This results in the linear differential equation

$$C\frac{dV}{dt} + \frac{V}{R} = 0.$$

Solving this equation for V yields the formula for exponential decay:

$$V(t) = V_0 e^{-\frac{t}{RC}} ,$$

where V_0 is the capacitor voltage at time $t = 0$.

The time required for the voltage to fall to $\frac{V_0}{e}$ is called the RC time constant and is given by

$$\tau = RC .$$

Complex impedance

The complex impedance, Z_C (in ohms) of a capacitor with capacitance *C* (in farads) is

$$Z_C = \frac{1}{sC}$$

The complex frequency *s* is, in general, a complex number,

$$s \ = \ \sigma + j\omega$$

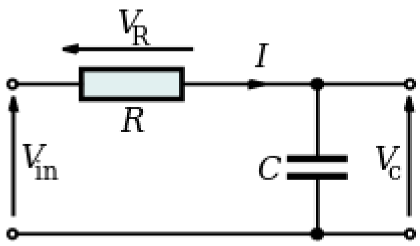
where

- j represents the imaginary unit:

$$j^2 = -1$$

- σ is the exponential decay constant (in radians per second), and
- ω is the sinusoidal angular frequency (also in radians per second).

Series circuit



Series RC circuit

By viewing the circuit as a voltage divider, the voltage across the capacitor is:

$$V_C(s) = \frac{1/Cs}{R + 1/Cs} V_{in}(s) = \frac{1}{1 + RCs} V_{in}(s)$$

and the voltage across the resistor is:

$$V_R(s) = \frac{R}{R + 1/Cs} V_{in}(s) = \frac{RCs}{1 + RCs} V_{in}(s)$$

Transfer functions

The transfer function from the input voltage to the voltage across the capacitor is

$$H_C(s) = \frac{V_C(s)}{V_{in}(s)} = \frac{1}{1 + RCs}.$$

Similarly, the transfer function from the input to the voltage across the resistor is

$$H_R(s) = \frac{V_R(s)}{V_{in}(s)} = \frac{RCs}{1 + RCs}.$$

Poles and zeros

Both transfer functions have a single pole located at

$$s = -\frac{1}{RC}.$$

In addition, the transfer function for the resistor has a zero located at the origin.

Gain and phase

The magnitude of the gains across the two components are:

$$G_C = |H_C(j\omega)| = \left|\frac{V_C(j\omega)}{V_{in}(j\omega)}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

and

$$G_R = |H_R(j\omega)| = \left|\frac{V_R(j\omega)}{V_{in}(j\omega)}\right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}},$$

and the phase angles are:

$$\phi_C = \angle H_C(j\omega) = \tan^{-1}(-\omega RC)$$

and

$$\phi_R = \angle H_R(j\omega) = \tan^{-1}\left(\frac{1}{\omega RC}\right).$$

These expressions together may be substituted into the usual expression for the phasor representing the output:

$$V_C \ = \ G_C V_{in} e^{j\phi_C}$$

$$V_R = G_R V_{in} e^{j\phi_R}.$$

Current

The current in the circuit is the same everywhere since the circuit is in series:

$$I(s) = \frac{V_{in}(s)}{R + \frac{1}{Cs}} = \frac{Cs}{1 + RCs} V_{in}(s)$$

Impulse response

The impulse response for each voltage is the inverse Laplace transform of the corresponding transfer function. It represents the response of the circuit to an input voltage consisting of an impulse or Dirac delta function.

The impulse response for the capacitor voltage is

$$h_C(t) = \frac{1}{RC} e^{-t/RC} u(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$

where $u(t)$ is the Heaviside step function and

$$\tau = RC$$

is the time constant.

Similarly, the impulse response for the resistor voltage is

$$h_R(t) = \delta(t) - \frac{1}{RC} e^{-t/RC} u(t) = \delta(t) - \frac{1}{\tau} e^{-t/\tau} u(t)$$

where $\delta(t)$ is the Dirac delta function

Frequency-domain considerations

These are frequency domain expressions. Analysis of them will show which frequencies the circuits (or filters) pass and reject. This analysis rests on a consideration of what happens to these gains as the frequency becomes very large and very small.

As $\omega \to \infty$ $G_C \to 0$ $\quad G_R \to 1$.

As $\omega \to 0$: $G_C \to 1 G_R \to 0$.This shows that, if the output is taken across the capacitor, high frequencies are attenuated (shorted to ground) and low frequencies are passed. Thus, the circuit behaves as a *low-pass filter*. If, though, the output is taken across the resistor, high frequencies are passed and low frequencies are attenuated (since the capacitor blocks the signal as its frequency approaches 0). In this configuration, the circuit behaves as a *high-pass filter*.

The range of frequencies that the filter passes is called its bandwidth. The point at which the filter attenuates the signal to half its unfiltered power is termed its cutoff frequency. This requires that the gain of the circuit be reduced to

$$G_C = G_R = \frac{1}{\sqrt{2}}.$$

Solving the above equation yields

$$\omega_c = \frac{1}{RC}$$

or

$$f_c = \frac{1}{2\pi RC}$$

which is the frequency that the filter will attenuate to half its original power.

Clearly, the phases also depend on frequency, although this effect is less interesting generally than the gain variations.

As $\omega \to 0$:

$$\phi_C \to 0$$

$$\phi_R \to 90^\circ = \pi/2^c.$$

As $\omega \to \infty$:

$$\phi_C \to -90^\circ = -\pi/2^c$$

$$\phi_R \to 0$$

So at DC (0 Hz), the capacitor voltage is in phase with the signal voltage while the resistor voltage leads it by 90°. As frequency increases, the capacitor voltage comes to have a 90° lag relative to the signal and the resistor voltage comes to be in-phase with the signal.

Time-domain considerations

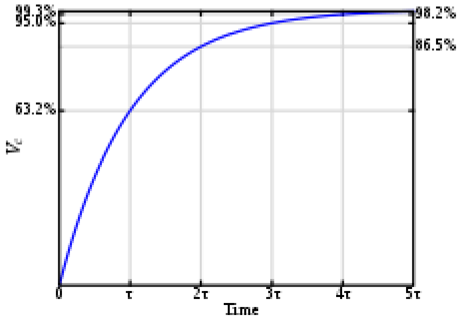
The most straightforward way to derive the time domain behaviour is to use the Laplace transforms of the expressions for V_C and V_R given above. This effectively transforms $j\omega \to s$. Assuming a step input (i.e. $V_{in} = 0$ before $t = 0$ and then $V_{in} = V$ afterwards):

$$V_{in}(s) = V\frac{1}{s}$$

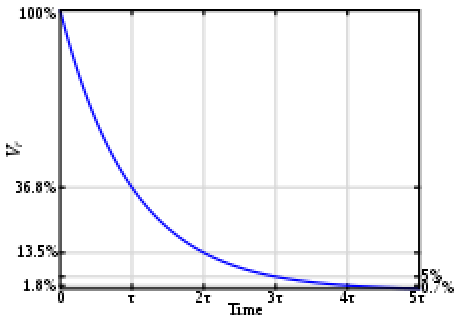
$$V_C(s) = V\frac{1}{1+sRC}\frac{1}{s}$$

and

$$V_R(s) = V\frac{sRC}{1+sRC}\frac{1}{s}.$$



Capacitor voltage step-response.



Resistor voltage step-response.

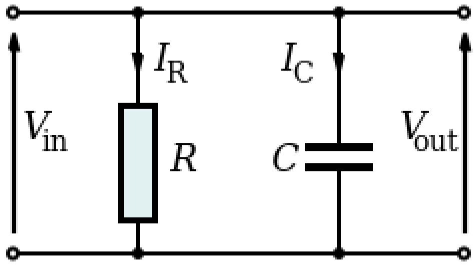
Partial fractions expansions and the inverse Laplace transform yield:

$$V_C(t) = V\left(1 - e^{-t/RC}\right)$$

$$V_R(t) = Ve^{-t/RC}.$$

These equations are for calculating the voltage across the capacitor and resistor respectively while the capacitor is charging; for discharging, the equations are vice-versa. These equations can be rewritten in terms of charge and current using the relationships C=Q/V and V=IR (see Ohm's law).

Parallel circuit



Parallel RC circuit

The parallel RC circuit is generally of less interest than the series circuit. This is largely because the output voltage V_{out} is equal to the input voltage V_{in} — as a result, this circuit does not act as a filter on the input signal unless fed by a current source.

With complex impedances:

$$I_R = \frac{V_{in}}{R}$$

and

$$I_C = j\omega C V_{in}.$$

This shows that the capacitor current is 90° out of phase with the resistor (and source) current. Alternatively, the governing differential equations may be used:

$$I_R = \frac{V_{in}}{R}$$

and

$$I_C = C\frac{dV_{in}}{dt}.$$

When fed by a current source, the transfer function of a parallel RC circuit is:

$$\frac{V_{out}}{I_{in}} = \frac{R}{1 + sRC}.$$

RL NETWORK SYNTHESIS:

A **resistor–inductor circuit** (**RL circuit**), or **RL filter** or **RL network**, is an electric circuit composed of resistors and inductors driven by a voltage or current source. A first order RL circuit is composed of one resistor and one inductor and is the simplest type of RL circuit.

The fundamental passive linear circuit elements are the resistor (R), capacitor (C) and inductor (L). These circuit elements can be combined to form an electrical circuit in four distinct ways: the RC circuit, the RL circuit, the LC circuit and the RLC circuit with the abbreviations indicating which components are used. These circuits exhibit important types of behaviour that are fundamental to analogue electronics. In particular, they are able to act as passive filters. This article considers the RL circuit in both series and parallel as shown in the diagrams.

In practice, however, capacitors (and RC circuits) are usually preferred to inductors since they can be more easily manufactured and are generally physically smaller, particularly for higher values of components.

Both RC and RL circuits form a single-pole filter. Depending on whether the reactive element (C or L) is in series with the load, or parallel with the load will dictate whether the filter is low-pass or high-pass.

Frequently RL circuits are used for DC power supplies to RF amplifiers, where the inductor is used to pass DC bias current and block the RF getting back into the power supply.

Complex impedance

The complex impedance Z_L (in ohms) of an inductor with inductance L (in henries) is

$$Z_L = Ls$$

The complex frequency s is a complex number,

$$s = \sigma + j\omega$$

where

- j represents the imaginary unit:

$$j^2 = -1$$

- $\sigma$is the exponential decay constant (in radians per second), and
- $\omega$is the angular frequency (in radians per second).

Eigenfunctions

The complex-valued eigenfunctions of *any* linear time-invariant (LTI) system are of the following forms:

$$\mathbf{V}(t) = \mathbf{A}e^{st} = \mathbf{A}e^{(\sigma+j\omega)t}$$
$$\mathbf{A} = Ae^{j\phi}$$
$$\Rightarrow \mathbf{V}(t) = Ae^{j\phi}e^{(\sigma+j\omega)t}$$
$$= Ae^{\sigma t}e^{j(\omega t+\phi)}$$

From Euler's formula, the real-part of these eigenfunctions are exponentially-decaying sinusoids:

$$v(t) = \mathrm{Re}\left\{V(t)\right\} = Ae^{\sigma t}\cos(\omega t + \phi)$$

Sinusoidal steady state

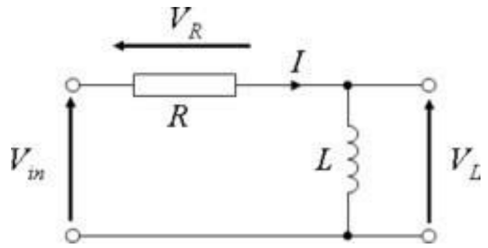
Sinusoidal steady state is a special case in which the input voltage consists of a pure sinusoid (with no exponential decay). As a result,

$$\sigma = 0$$

and the evaluation of *s* becomes

$$s = j\omega$$

Series circuit



Series RL circuit

By viewing the circuit as a voltage divider, we see that the voltage across the inductor is:

$$V_L(s) = \frac{Ls}{R + Ls}V_{in}(s)$$

and the voltage across the resistor is:

$$V_R(s) = \frac{R}{R + Ls}V_{in}(s)$$

Current

The current in the circuit is the same everywhere since the circuit is in series:

$$I(s) = \frac{V_{in}(s)}{R + Ls}$$

Transfer functions

The transfer function for the inductor is

$$H_L(s) = \frac{V_L(s)}{V_{in}(s)} = \frac{Ls}{R + Ls} = G_L e^{j\phi_L}$$

Similarly, the transfer function for the resistor is

$$H_R(s) = \frac{V_R(s)}{V_{in}(s)} = \frac{R}{R + Ls} = G_R e^{j\phi_R}$$

Poles and zeros

Both transfer functions have a single pole located at

$$s = -\frac{R}{L}$$

In addition, the transfer function for the inductor has a zero located at the origin.

Gain and phase angle

The gains across the two components are found by taking the magnitudes of the above expressions:

$$G_L = |H_L(s)| = \left|\frac{V_L(s)}{V_{in}(s)}\right| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

and

$$G_R = |H_R(s)| = \left|\frac{V_R(s)}{V_{in}(s)}\right| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

and the phase angles are:

$$\phi_L = \angle H_L(s) = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

and

$$\phi_R = \angle H_R(s) = \tan^{-1}\left(-\frac{\omega L}{R}\right)$$

Phasor notation

These expressions together may be substituted into the usual expression for the phasor representing the output:

$$V_L = G_L V_{in} e^{j\phi_L}$$
$$V_R = G_R V_{in} e^{j\phi_R}$$

Impulse response

The impulse response for each voltage is the inverse Laplace transform of the corresponding transfer function. It represents the response of the circuit to an input voltage consisting of an impulse or Dirac delta function.

The impulse response for the inductor voltage is

$$h_L(t) = \delta(t) - \frac{R}{L}e^{-t\frac{R}{L}}u(t) = \delta(t) - \frac{1}{\tau}e^{-\frac{1}{\tau}t}u(t)$$

where $u(t)$ is the Heaviside step function and

$$\tau = \frac{L}{R}$$

is the time constant.

Similarly, the impulse response for the resistor voltage is

$$h_R(t) = \frac{R}{L}e^{-t\frac{R}{L}}u(t) = \frac{1}{\tau}e^{-\frac{1}{\tau}t}u(t)$$

Zero input response (ZIR)

The Zero input response, also called the natural response, of an RL circuit describes the behavior of the circuit after it has reached constant voltages and currents and is disconnected from any power source. It is called the zero-input response because it requires no input.

The ZIR of an RL circuit is:

$$i(t) = i(0)e^{-\frac{R}{L}t} = i(0)e^{-\frac{1}{\tau}t}$$

Frequency domain considerations

These are frequency domain expressions. Analysis of them will show which frequencies the circuits (or filters) pass and reject. This analysis rests on a consideration of what happens to these gains as the frequency becomes very large and very small.

As $\omega \to \infty$:

$$G_L \to 1$$
$$G_R \to 0$$

As $\omega \to 0$:

$$G_L \to 0$$
$$G_R \to 1$$

This shows that, if the output is taken across the inductor, high frequencies are passed and low frequencies are attenuated (rejected). Thus, the circuit behaves as a *high-pass filter*. If, though, the output is taken across the resistor, high frequencies are rejected and low frequencies are passed. In this configuration, the circuit behaves as a *low-pass filter*. Compare this with the behaviour of the resistor output in an RC circuit, where the reverse is the case.

The range of frequencies that the filter passes is called its bandwidth. The point at which the filter attenuates the signal to half its unfiltered power is termed its cutoff frequency. This requires that the gain of the circuit be reduced to

$$G_L = G_R = \frac{1}{\sqrt{2}}.$$

Solving the above equation yields

$$\omega_c = \frac{R}{L} \text{ rad/s}$$

or

$$f_c = \frac{R}{2\pi L} \text{ Hz}$$

which is the frequency that the filter will attenuate to half its original power.

Clearly, the phases also depend on frequency, although this effect is less interesting generally than the gain variations.

As $\omega \to 0$:

$$\phi_L \to 90^\circ = \frac{\pi}{2}^c$$
$$\phi_R \to 0$$

As $\omega \to \infty$:

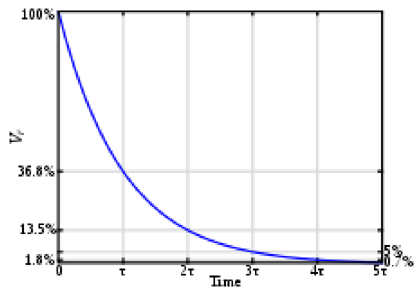
$$\phi_L \to 0$$
$$\phi_R \to -90^\circ = -\frac{\pi}{2}^c$$

So at DC (0 Hz), the resistor voltage is in phase with the signal voltage while the inductor voltage leads it by 90°. As frequency increases, the resistor voltage comes to have a 90° lag relative to the signal and the inductor voltage comes to be in-phase with the signal.

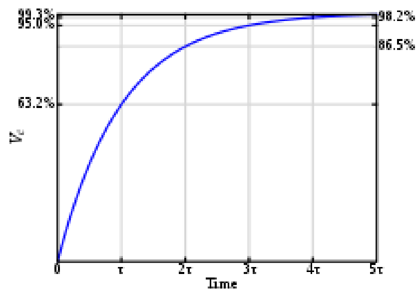
Time domain considerations

The most straightforward way to derive the time domain behaviour is to use the Laplace transforms of the expressions for V_L and V_R given above. This effectively transforms $j\omega \to s$. Assuming a step input (i.e., $V_{in} = 0$ before $t = 0$ and then $V_{in} = V$ afterwards):

$$V_{in}(s) = V\frac{1}{s}$$
$$V_L(s) = V\frac{sL}{R + sL}\frac{1}{s}$$
$$V_R(s) = V\frac{R}{R + sL}\frac{1}{s}$$



Inductor voltage step-response.



Resistor voltage step-response.

RLC NETWORK SYNTHESIS:

A series RLC circuit: a resistor, inductor, and a capacitor

An **RLC circuit** (the letters R, L and C can be in other orders) is an electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series or in parallel. The RLC part of the name is due to those letters being the usual electrical symbols for resistance, inductance and capacitance respectively. The circuit forms a harmonic oscillator for current and will resonate in a similar way as an LC circuit will. The main difference that the presence of the resistor makes is that any oscillation induced in the circuit will die away over time if it is not kept going by a source. This effect of the resistor is called damping. The presence of the resistance also reduces the peak resonant frequency somewhat. Some resistance is unavoidable in real circuits, even if a resistor is not specifically included as a component. An ideal, pure LC circuit is an abstraction for the purpose of theory.

There are many applications for this circuit. They are used in many different types of oscillator circuits. Another important application is for tuning, such as in radio receivers or television sets, where they are used to select a narrow range of frequencies from the ambient radio waves. In this role the circuit is often referred to as a tuned circuit. An RLC circuit can be used as a band-pass filter, band-stop filter, low-pass filter or high-pass filter. The tuning application, for instance, is an example of band-pass filtering. The RLC filter is described as a *second-order* circuit, meaning that any voltage or current in the circuit can be described by a second-order differential equation in circuit analysis.

The three circuit elements can be combined in a number of different topologies. All three elements in series or all three elements in parallel are the simplest in concept and the most straightforward to analyse. There are, however, other arrangements, some with practical importance in real circuits. One issue often encountered is the need to take into account inductor resistance. Inductors are typically constructed from coils of wire, the resistance of which is not usually desirable, but it often has a significant effect on the circuit.

Resonance

An important property of this circuit is its ability to resonate at a specific frequency, the resonance frequency, f_0. Frequencies are measured in units of hertz. In this article, however, angular frequency, ω_0, is used which is more mathematically convenient. This is measured in radians per second. They are related to each other by a simple proportion,

$$\omega_0 = 2\pi f_0$$

Resonance occurs because energy is stored in two different ways: in an electric field as the capacitor is charged and in a magnetic field as current flows through the inductor. Energy can be transferred from one to the other within the circuit and this can be oscillatory. A mechanical analogy is a weight suspended on a spring which will oscillate up and down when released. This is no passing metaphor; a weight on a spring is described by exactly the same second order differential equation as an RLC circuit and for all the properties of the one system there will be found an analogous property of the other. The mechanical property answering to the resistor in the circuit is friction in the spring/weight system. Friction will slowly bring any oscillation to a halt if there is no external force driving it. Likewise, the resistance in an RLC circuit will "damp" the oscillation, diminishing it with time if there is no driving AC power source in the circuit.

The resonance frequency is defined as the frequency at which the impedance of the circuit is at a minimum. Equivalently, it can be defined as the frequency at which the impedance is purely real (that is, purely resistive). This occurs because the impedances of the inductor and capacitor at resonance are equal but of opposite sign and cancel out. Circuits where L and C are in parallel rather than series actually have a maximum impedance rather than a minimum impedance. For this reason they are often described as antiresonators, it is still usual, however, to name the frequency at which this occurs as the resonance frequency.

Natural frequency

The resonance frequency is defined in terms of the impedance presented to a driving source. It is still possible for the circuit to carry on oscillating (for a time) after the driving source has been removed or it is subjected to a step in voltage (including a step down to zero). This is similar to the way that a tuning fork will carry on ringing after it has been struck, and the effect is often called ringing. This effect is the peak natural resonance frequency of the circuit and in general is not exactly the same as the driven resonance frequency, although the two will usually be quite close to each other. Various terms are used by different authors to distinguish the two, but resonance frequency unqualified usually means the driven resonance frequency. The driven frequency may be called the undamped resonance frequency or undamped natural frequency and the peak frequency may be called the damped resonance frequency or the damped natural frequency. The reason for this terminology is that the driven resonance frequency in a series or parallel resonant circuit has the value[1]

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

This is exactly the same as the resonance frequency of an LC circuit, that is, one with no resistor present. The resonant frequency for an RLC circuit is the same as a circuit in which there is no damping, hence undamped resonance frequency. The peak resonance frequency, on the other hand, depends on the value of the resistor and is described as the damped resonant frequency. A highly damped circuit will fail to resonate at all when not driven. A circuit with a value of resistor that causes it to be just on the edge of ringing is called critically damped. Either side of critically damped are described as underdamped (ringing happens) and overdamped (ringing is suppressed).

Damping

Damping is caused by the resistance in the circuit. It determines whether or not the circuit will resonate naturally (that is, without a driving source). Circuits which will resonate in this way are described as underdamped and those that will not are overdamped. Damping attenuation (symbol α) is measured in nepers per second. However, the unitless damping factor (symbol ζ, zeta) is often a more useful measure, which is related to α by

$$\zeta = \frac{\alpha}{\omega_0}$$

The special case of $\zeta = 1$ is called critical damping and represents the case of a circuit that is just on the border of oscillation. It is the minimum damping that can be applied without causing oscillation.

Bandwidth

The resonance effect can be used for filtering, the rapid change in impedance near resonance can be used to pass or block signals close to the resonance frequency. Both band-pass and band-stop filters can be constructed and some filter circuits are shown later in the article. A key parameter in filter design is bandwidth. The bandwidth is measured between the 3dB-points, that is, the frequencies at which the power passed through the circuit has fallen to half the value passed at resonance. There are two of these half-power frequencies, one above, and one below the resonance frequency

$$\Delta\omega = \omega_2 - \omega_1$$

where $\Delta\omega$ is the bandwidth, ω_1 is the lower half-power frequency and ω_2 is the upper half-power frequency. The bandwidth is related to attenuation by,

$$\Delta\omega = 2\alpha$$

when the units are radians per second and nepers per second respectively[*citation needed*]. Other units may require a conversion factor. A more general measure of bandwidth is the fractional bandwidth, which expresses the bandwidth as a fraction of the resonance frequency and is given by

$$F_b = \frac{\Delta\omega}{\omega_0}$$

The fractional bandwidth is also often stated as a percentage. The damping of filter circuits is adjusted to result in the required bandwidth. A narrow band filter, such as a notch filter, requires low damping. A wide band filter requires high damping.

Q factor

The Q factor is a widespread measure used to characterise resonators. It is defined as the peak energy stored in the circuit divided by the average energy dissipated in it per radian at resonance. Low *Q* circuits are therefore damped and lossy and high *Q* circuits are underdamped. *Q* is related to bandwidth; low *Q* circuits are wide band and high *Q* circuits are narrow band. In fact, it happens that *Q* is the inverse of fractional bandwidth

$$Q = \frac{1}{F_b} = \frac{\omega_0}{\Delta\omega}$$

Q factor is directly proportional to selectivity, as Q factor depends inversely on bandwidth.

Applications

Variable tuned circuits

A very frequent use of these circuits is in the tuning circuits of analogue radios. Adjustable tuning is commonly achieved with a parallel plate variable capacitor which allows the value of C to be changed and tune to stations on different frequencies. For the IF stage in the radio where the tuning is preset in the factory the more usual solution is an adjustable core in the

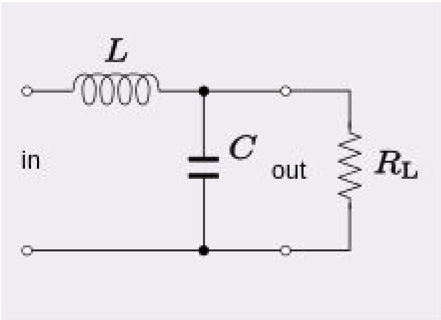


Fig. RLC circuit as a low-pass filter

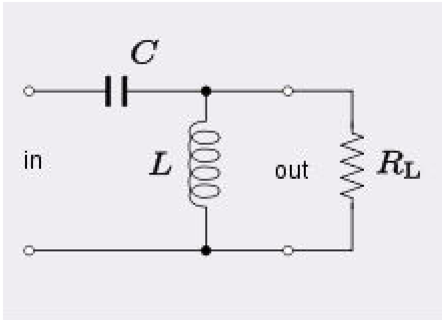


Fig. RLC circuit as a high-pass filter

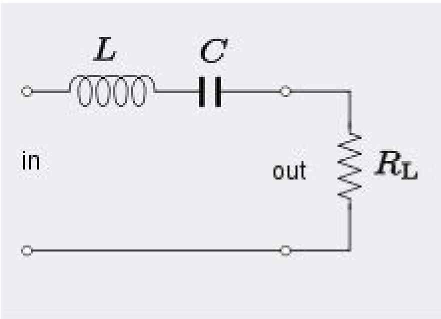


Fig. RLC circuit as a series band-pass filter in series with the line

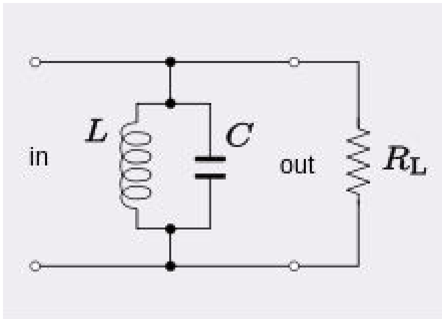


Fig. RLC circuit as a parallel band-pass filter in shunt across the line

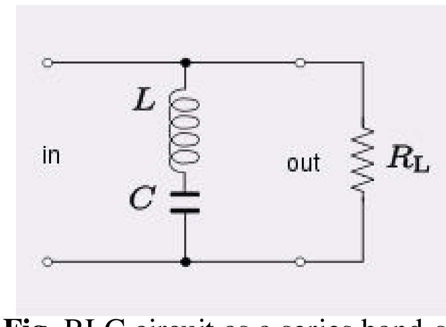


Fig. RLC circuit as a series band-stop filter in shunt across the line

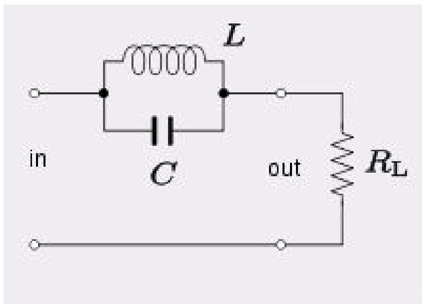


Fig. RLC circuit as a parallel band-stop filter in series with the line

inductor to adjust L. In this design the core (made of a high permeability material that has the effect of increasing inductance) is threaded so that it can be screwed further in, or screwed further out of the inductor winding as required.

FOSTER'S REACTANCE THEOREM:

Introduction:

Network synthesis involves the methods used to determine an electric circuit that satisfy certain specifications. Given an impulse response there are many techniques that can be used to synthesize a circuit with the specified response. Different methods may also be used to synthesize circuits, all of which may be optimal. Hence the solution to a network synthesis problem is never unique.

Many applications today use digital processing in lieu of analog processing and the GHz spectrum is finding increasing use in applications such as wireless communications. However, operation at high frequencies requires analog filtering and processing circuits simply because using digital techniques is neither realistic nor economical. Another advantage that analog devices have over their digital counterparts is their ability to operate with wide instantaneous bandwidths and moderately high dynamic ranges at microwave frequencies.

For a **Foster** 1 realisation the component values are given by the partial fraction expansion

$$Z(s) = K_{inf}s + \frac{K_0}{s} + \frac{K_1 s}{s^2 + w_1^2} + \frac{K_2 s}{s^2 + w_2^2} + \cdots + \frac{K_n s}{s^2 + w_n^2}$$

While for **the Foster 2** form the values are given by the alternative partial fraction expansion

$$Y(s) = K'_{inf}s + \frac{K'_0}{s} + \frac{K'_1 s}{s^2 + w_1'^2} + \frac{K'_2 s}{s^2 + w_2'^2} + \cdots + \frac{K'_n s}{s^2 + w_n'^2}$$

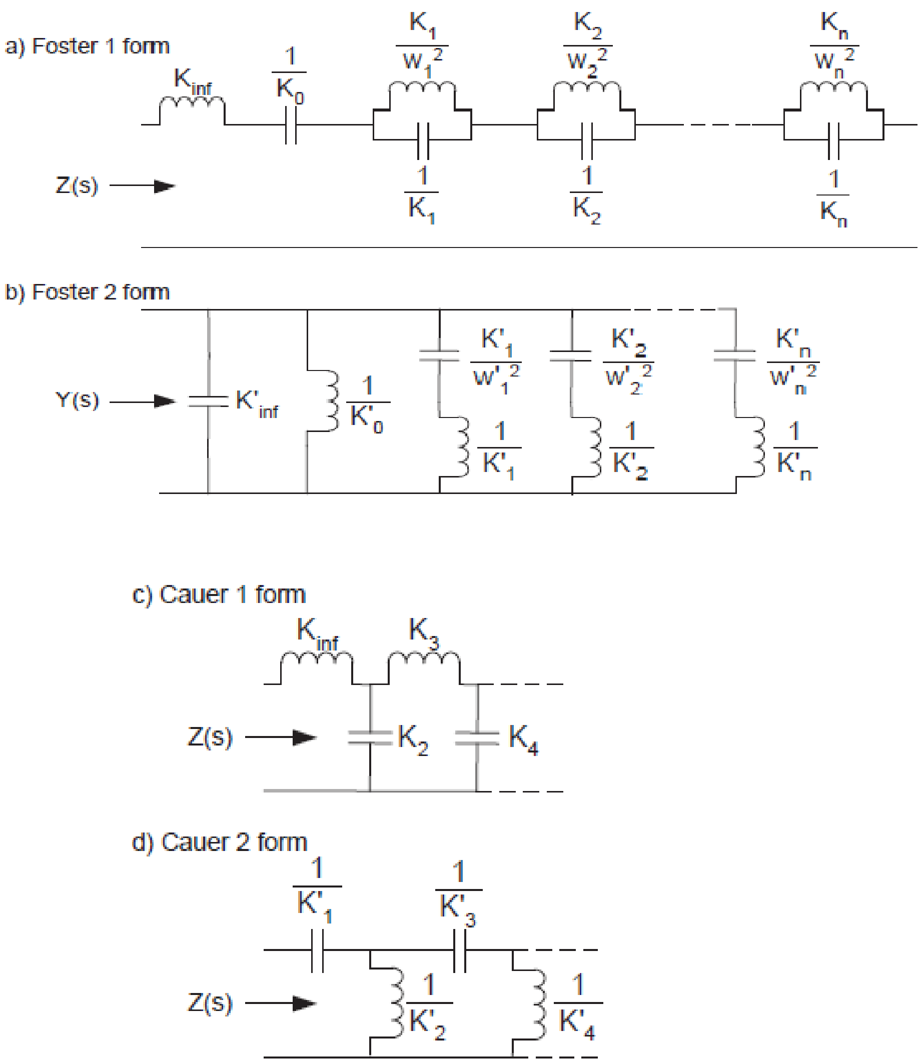
For the **Cauer 1** realization the component values are given by a continued fraction expansion around infinity

$$Z(s) = K_{inf}s + \cfrac{1}{K_2 s + \cfrac{1}{K_3 s + \cdots}}$$

The **Cauer 2** values are given by a continued fraction expansion around zero

$$Z(s) = \frac{K'_1}{s} + \cfrac{1}{\frac{K'_2}{s} + \cfrac{1}{\frac{K'_3}{s} + \cdots}}$$

Foster and Cauer network realisations. These allow simple determination of the Required component values by continued and partial fraction expansions



Foster's reactance theorem is an important theorem in the fields of electrical network analysis and synthesis. The theorem states that the reactance of a passive, lossless two-terminal (one-port) network always strictly monotonically increases with frequency. It is easily seen that the reactances of inductors and capacitors individually increase with frequency and from that basis a proof for passive lossless networks generally can be constructed. The proof of the theorem was presented by Ronald Martin Foster in 1924,although the principle had been published earlier by Foster's colleagues at American Telephone & Telegraph.

The theorem can be extended to admittances and the encompassing concept of immittances. A consequence of Foster's theorem is that poles and zeroes of the reactance must alternate with frequency. Foster used this property to develop two canonical forms for realising these networks. Foster's work was an important starting point for the development of network synthesis.

It is possible to construct non-Foster networks using active components such as amplifiers. These can generate an impedance equivalent to a negative inductance or capacitance. The negative impedance converter is an example of such a circuit.

Conclusion:

By following the step by step procedure in each form the desired networks are synthesised. The following are the circuit diagrams obtained as a result of the analysis

Reference:

[1].Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi,2000
[2]. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education 2004
[3]. Paranjothi SR, "Electric Circuits Analysis," New Age International Ltd., New Delhi, 1996.

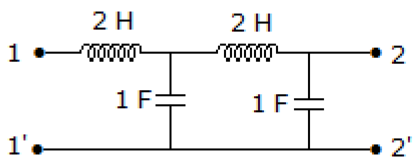
Post Test MCQs:

1.

The transfer function $\frac{s}{s+a}$ is for

a.. low pass filter
b. notch filter
c. high pass filter
d. band pass filter

2. For the ladder network of figure, open circuit driving point impedance at port 1 =



a.

$$2s + \cfrac{1}{s + \cfrac{1}{2s + \cfrac{1}{s}}}$$

b.

$$2s + \cfrac{1}{2s + \cfrac{1}{s + \cfrac{1}{2s}}}$$

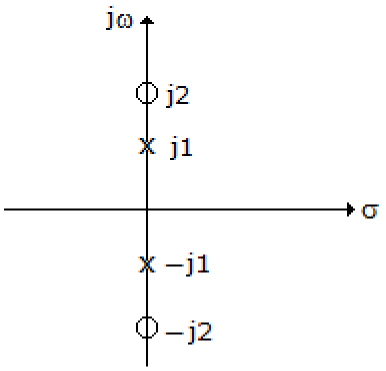
c.

$$s + \cfrac{1}{2s + \cfrac{1}{s + \cfrac{1}{2s}}}$$

d.

$$s + \cfrac{1}{2s + \cfrac{1}{s + \cfrac{1}{2s}}}$$

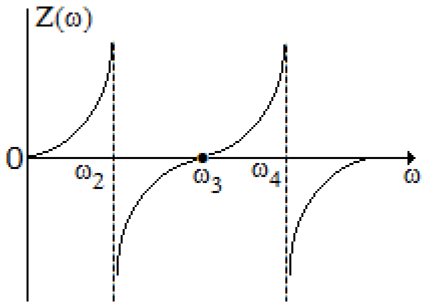
3. A pole zero plot of a filter is shown in figure. It is



a low pass filter
b. high pass filter
c. band pass filter
d. all pass filter

4. An RC impedance function has a pole at $s = 0$. The first element in the Foster form of synthesis
a. is R
b..may be R and C
c. is C
d. is R and C in parallel

5. Figure shows the variation of Z(ω) for a positive real function. The function is



a. .RC impedance
b. RL impedance
c. RLC impedance
d. Reactance function